ARITHMETIC IN PRIMARY SCHOOLS

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ARITHMETIC IN PRIMARY SCHOOLS

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ARITHMETIC IN PRIMARY SCHOOLS

A REPORT ON THE TEACHING OF ARITHMETIC
AND SPATIAL KNOWLEDGE IN PRIMARY SCHOOLS

PREPARED BY THE MATHEMATICS SECTION OF THE
METROPOLITAN BRANCH OF THE ASSOCIATION OF
TEACHERS IN COLLEGES AND DEPARTMENTS OF
EDUCATION

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FOREWORD

by Professor H. R. HAMLEY, M.A., M.Sc.,
Institute of Education, University of London

This comprehensive and authoritative report on the teaching of mathematics in Primary Schools was prepared in response to a widely felt need. It is true that some excellent books on the subject have been published in America but most of them are unsuitable for English readers partly because they are unnecessarily long and detailed and partly because they presuppose experience of American conditions. The present volume supplies just what is needed to help the inquiring teacher in this country and to stimulate him to adventurous teaching. The purpose of the Report is not to show in detail how the fundamental processes should be taught but how the whole content of the subject may be enriched. Occasionally the reader is presented with a challenge, as when he is told, for example, that inquiries are urgently needed before a syllabus in elementary mathematics can be drawn up that will take into account the child's mental development and his natural interests.

The Committee have wisely put first things first; they have, for example, realized that what is fundamental in elementary mathematics is not the fundamental process itself, but the concept behind the process. So we find that considerable emphasis has been placed on "units," for it is only through a clear understanding of physical units that concepts such as length, area and volume can be fully appreciated. It comes as a surprise, even to intelligent children, to realize that a cubic inch of sand when emptied from an inch cube and spread over a sheet of paper is still the same cubic inch. There is no doubt that our next advance in the teaching of elementary mathematics will come not from perfections of method but from a re-examination of the logical and psychological foundations of the subject.

To most teachers the sections of the Report which deal with Spatial Knowledge will be found interesting and fresh. These have been based upon the definite results of research. Although the suggestions have the backing of that authority, they should be received as "suggestions." It would be most unfortunate if the enthusiastic teacher were to make this part of his teaching as stereotyped as the teaching of arithmetic has become in many schools. The suggestions of the Committee are intended to free the mind of the teacher, not to keep him in closer bondage.

The Report will commend itself to teachers and heads of schools who believe that mathematics in schools has cultural value in addition to its value to the potential consumer.

ARITHMETIC IN PRIMARY SCHOOLS

PREFACE TO THE SECOND EDITION

The First Edition of this Report appeared in 1940 at the end of that ominous period of inactivity which puzzled Britain at the beginning of the last war. Within a few weeks of its publication all the stocks in the publishers' hands were destroyed in the blitz on London on 30th December 1940. Yet sufficient copies had been sold to create a demand which has persisted during the years that the Report has been out of print. It has therefore seemed right to prepare a new issue. But many changes have made revision necessary. The Education Act of 1944 has altered the structure of the School system. The Training College Association which sponsored the Report has been reconstituted with greatly extended membership and responsibilities as the Association of Teachers in Colleges and Departments of Education. Only three members of the original Committee which wrote the Report are still concerned with the training of teachers. Yet the number of students now in Training Colleges, or waiting for admission, is much greater than ever before, and we believe that this Report, produced in the first instance for "the consideration of teachers, students in training and others who are interested . . . in the teaching of arithmetic," will help to guide and stimulate these thousands who are pondering on the aims and methods of teaching the fundamentals of arithmetic.

E. R. HAMILTON, *Borough Road Training College*
A. PAGE, *Borough Road Training College*
E. M. WILLIAMS, *City of Leicester Training College*

on behalf of THE METROPOLITAN BRANCH OF THE ASSOCIATION OF TEACHERS IN COLLEGES AND DEPARTMENTS OF EDUCATION.

December, 1946

The following were members of the Committee appointed by the Metropolitan Branch of the Training College Association* to draw up the first edition of this Report:

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CONTENTS

PAGE

FOREWORD BY PROFESSOR H. R. HAMLEY, M.A., M.Sc. v

INTRODUCTION xi

CHAPTER

I. NUMBER AND SPATIAL KNOWLEDGE BEFORE JUNIOR SCHOOL AGE 1

II. SPATIAL KNOWLEDGE 8

III. THE FOUR RULES IN NUMBER 15

IV. UNITS OF MEASUREMENT 24

V. FURTHER CONTENTS OF ARITHMETIC SYLLABUS . 39

VI. METHODS OF TEACHING 46

APPENDIX

I. THE BRITISH MEASURES: HISTORICAL NOTES . 56

II. THE SELECTION EXAMINATION AT 11 . . . 69

III. BIBLIOGRAPHY 75

INTRODUCTION

This memorandum is intended for the consideration of teachers, students in training and others who are interested in the changes now taking place in the teaching of arithmetic in schools. There was a time when the contents of the course to be followed in elementary schools and the standard of work to be expected in each year were prescribed and each teacher was familiar with them. A new generation of teachers knows nothing of the old "standards" and finds nothing definite to put in their place. Thoughtful and progressive teachers everywhere confess that the one subject of the curriculum that causes them anxiety is arithmetic. There are several causes which have helped to bring about this state of uncertainty.

In the first place there is the recognized need to think of all the subjects of the Primary School curriculum in terms of activity. This means that "arithmetic" must no longer be merely the manipulation of number or money and the learning of tables of length, weight, time, etc., but must include dealing with situations which need number operations, carrying out money transactions, and actual weighing and measuring. This involves a new type of organization of the arithmetic lesson and has raised many questions in the minds of able teachers and of those whose duty it is to help young students through their first attempts to allow their classes such activities.

It is now generally accepted that the subject which most frequently causes the children themselves anxiety and robs them of self-confidence is arithmetic. Too often they are bored by it and find in it no relevance to the stirring life outside school or the many pleasurable occupations that school itself can offer. The problem of presenting arithmetic so that it is seen by the children to be essential to some of their interests and indeed interesting in itself is no easy one to solve.

The arithmetic that was taught in the "standards" of the elementary schools included most of the numerical operations that were required in trade and commerce in those days but many of these, such as calculations of certain areas, of compound interest and of Stock Exchange figures, were specialized and did not correspond to the needs of the majority of the children. They were often mechanical skills which were never used in later life and brought little understanding of the sphere in which they had their specialized use. In the modern world with ever greater specialization such calculations are of still less significance to the majority of citizens and it is recognized that ready reckoners and calculating machines can be used for the heavier computations which are still necessary. This reduction in the amount of arithmetical skill required of the adult makes it possible for the Primary School, which lays the foundations of this skill, to reduce its demands on the children and release them from the tension which over-pressure has so often caused. At the same time it can aim at a high standard of accuracy in the number processes that are actually needed.

Facility in number operations is now seen to be only one of the aims, and not the most important, of arithmetic teaching; without numerical instances description is insufficient for a full understanding of many of the phenomena, whether natural or man-made, of the world to-day. Quantitative comparison, precise measurement and records of observations are necessary before we reach a full realization of the nature of some events; e.g. the observation that a district is being rapidly covered with houses is much less fruitful than a study of the figures for several years showing the number of houses, population, local government expenses, and the rateable value of the district in their relationship with one another; similarly the apparent movement of the sun in the sky is only vaguely understood until precise measurements of the lengths and directions of shadows have been made.[1] Experiment alone can determine how much of this interpretative arithmetic can be undertaken with children under eleven. This memorandum endeavours to set out lines along which such experiment could usefully be made.

In the past "arithmetic" has been almost entirely confined to number. The properties of space have been neglected with the exception of measurement of length and the areas of simple shapes. Size is not the only spatial property of importance. Shape, position, direction are ideas of practical value and easily within the understanding of Primary School children. Unfortunately the word geometry has come to mean the formal logical treatment of spatial properties and its use in this pamphlet might have led to misunderstanding. We have therefore referred to "spatial knowledge" as showing more clearly the kind of geometry that we believe should find a place in Primary Schools. Its value lies not in its practical use alone but in the sense of power it gives to many children who can understand easily the visual forms of practical geometry while highly abstract number work presents them with real difficulty. It is because we attach so much importance to the teaching of spatial properties that the chapter concerned is placed so early in the Report.

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[1] See Chapter II.

In forming curricula for schools in the past attention seems to have been focused on the full course of study suited to an adult without any regard to its suitability for children. This was certainly true of the teaching of geometry. In arithmetic the courses of study appear to have been taken with little change from the commercial schools of the eighteenth century. If the end of the arithmetic course was to be the power to perform the various calculations of the merchant, financier or surveyor, then working backwards the necessary achievement of each standard in school could be determined so that the whole study should be a logical development. To-day this way of drawing up a syllabus is seen to be psychologically unsound. Attempts are now being made to discover the minimum age at which a child can learn any process in arithmetic with the least waste of time and effort. In America the Committee of Seven of the Northern Illinois Conference on Supervision have been working for a number of years on this question and their findings help us to decide when an arithmetical topic can be most effectively taught. They have discovered, for example, that children master long division with less strain and with greater efficiency at a mental age of about twelve years than at an earlier stage. For this reason it is suggested that long division in the full range of numbers should not be expected of any but the ablest children in the Primary School and that compound long division should be postponed to the post-primary stage.[1] Since an investigation carried out in America cannot give all the information needed in English schools with our different coinage and units of measurement, similar inquiries are urgently needed in this country before an arithmetic syllabus, based on an accurate knowledge of the age at which children are ready for the various processes and on the incentives available at different ages, can be drawn up with a scientific degree of certainty.

One of the chief difficulties of the teacher of arithmetic is the wide range of specific arithmetical ability even among a class selected on the grounds of general ability. In this memorandum we have considered the pupils as falling into two types: the A type who find definite intellectual pleasure in solving mathematical problems of an abstract kind and in developing a sense of mastery in a particular field of mathematical skill: the B type for whom mathematics has significance only when it is kept in close touch with practical situations which they can comprehend. It has been the practice to provide the same sort of mathematical education for both types with a syllabus largely dominated by the needs and interests of those of more than average ability. Since the B type of pupil can only appreciate mathematics when it is seen as a tool to solve practical problems in which he is interested, this scheme has resulted in unreality and boredom. We believe that the interests of *both* types of pupil are best served by making the utilitarian aim paramount, though not unique, in arithmetic throughout the Junior School. For the A class no harm will be done by allowing them to carry their manipulative skill beyond the range of their practical interests but this additional abstract work should not be regarded as part of the normal work required of children under 11 nor should it be expected in the Entrance Examination. In this memorandum we have attempted to set out the amount and kind of arithmetic that all *normal* children should be able to master by the age of eleven. Perhaps experience will show that we are even now expecting too much; we are convinced that it is at least sufficient as a basis for post-primary work even for children going on to Secondary Grammar Schools.

In the light of preceding paragraphs it must be clear that we believe that active experience must be the basis of children's learning in arithmetic as in other subjects. Such activity is generally recognized as essential to number work in the Infant School and we recommend that formal memorizing and the working of sums divorced from actual situations should not be expected of Infant children though we are aware that a *few* gifted children will wish to do written sums at an early age. We strongly urge that activity in measuring, weighing, fitting and shopping should *continue* to be an essential part of the arithmetic course throughout the *Primary* School and that children should be led to regard the recording of such experiences as of equal importance with the working of "sums." Indeed, they may well regard it as more important, since they can see the useful results of their endeavours. Much of this practical work will arise in connection with interests in handwork, history, geography, or a study of environment, and will gain in significance when this is so.

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[1] See reference to C. Washburne in Appendix III.

Children who have frequent opportunities of weighing, measuring and shopping, will often improvise units for comparison, for example handbreadths in length, marbles for money, mugs for capacity. This will lead them to an understanding of the need for standardized units. Children are apt to accept the English measures and money as a mechanical custom. Before the age of eleven they use these measures in a very restricted way, and the lessons and examples set tend to become artificial exercises. This can be remedied, to a certain extent, by giving the children a simple knowledge of the vital importance of these units to the people who introduced them. Most children of this age are interested in stories about people, and the way in which they overcame their difficulties. The present-day English measures and money have a long and entertaining history of the kind that English children will understand and remember. Appendix I of this Report refers briefly to the history of some of our measures and coins.

Specially devised apparatus is still required after children leave the Infant School, particularly in connection with memorizing tables; spatial work, too, needs its own special tools; but we strongly advocate, both in number work and in geometry, the use of everyday materials and home-made appliances. The more closely this material is related to what is used in home, shop, or street the more clearly the children see the applications of the process they are learning. The use of elaborate symbols and number patterns may confuse rather than clarify since they divert the children's interest from the numbers themselves.

It is frequently said that the standards of accuracy in calculation have deteriorated in recent years. If this is true only of difficult computation that can be performed by machine or ready reckoner we need feel no alarm; but it is essential that all normal children should reach a high standard in the simpler processes. We therefore recommend that children should have constant and varied drill in all the basic facts of addition, subtraction, multiplication and division so that these are known absolutely. Properly spaced over the first two years in the Junior School all these tables can be efficiently learnt without imposing any formal memorizing on the Infant School.

The criticism may be made that the scope of the work suggested in this report is inadequate as a preparation for the Examination for selection for Secondary Schools. Detailed consideration of this view is given in Appendix II. We would emphasize that we do not suggest restriction of the abler children but in view of the fact that the entire Junior School population is tested for admission to appropriate Secondary Schools we should like to see a general reduction in the scope and difficulty of the questions set in some districts. It is a matter of common experience that many entrants to Secondary Grammar Schools have already attempted more arithmetic than their new schools assume to be known and much has to be learnt again. If even these gifted children will not suffer from a reduction in the amount of arithmetic to be learnt before eleven there seems to be no impediment but inertia to the general reduction in the Primary School arithmetic syllabus with a corresponding widening of its interest and a consequent improvement in its educative value.

CHAPTER I

NUMBER AND SPATIAL KNOWLEDGE BEFORE JUNIOR SCHOOL AGE

Any attempt to frame a course of arithmetic for Junior School children must be preceded by an examination of the knowledge and skill that they can acquire by seven years of age through their natural interests and activities both at home and in school.

CHILDREN UNDER FIVE.

Ideas of number and space develop early through play with the varied material that ordinary homes supply. Children hear such words as "much," "few," "long," "heavy," "full," and can generally use them correctly by the time they enter the Infant School at the age of five. Direct teaching is undesirable during this first period. It is sufficient for children to be given ample material for "home" play, e.g. dough, water, sand, bricks, beads, etc. They will then compare, measure and weigh spontaneously, thus laying a firm foundation in experience for the formal work that must come later. Some children will begin to count and will pick up the names of the first few numbers from people round them but an attempt to force this knowledge will not help a child's later progress.

THE BASIS OF ARITHMETIC IN THE INFANT SCHOOL.

On entering the Infant School at five children still need plenty of space and material for similar kinds of play; where the home has failed to provide it in the earlier years it will be the more necessary in school. Group occupations in the wider social environment will necessitate counting. Co-operation now makes large constructional undertakings also possible and the interest of five-year-old children in the life of the home and in powerful moving things will show itself in setting tables, building engines, buses, aeroplanes, etc. This type of play will extend counting and involve comparison of sizes and shapes; later, as shop interests develop, opportunities for handling toy money will occur. The first experiences of addition and subtraction can well come now before any attempt is made to express the processes in number symbols. As at all stages, the child's grasp and use of the language of number should be in advance of his knowledge of figures. We would emphasize the importance of those early activities where the basic operation of counting ensures that the child understands the nature of numbers. Many of the errors of older children arise from the fact that they were taught to write and manipulate *figures* rather than to appreciate *number*. While it is agreed that the symbols for numbers and the nature of the four operations of adding, taking away, grouping and sharing should be known at the end of the Infant School period, it is urged that living out experiences in which number is important and playing games which depend in part on number for their enjoyment are the two chief means by which Infant School children achieve that mastery over number work which should precede skill with written numbers and should not at this stage give place to it. It is appreciated that many L.E.A.'s recognize that such work cannot be done without adequate materials and space, and that where such provision is insufficient and classroom conditions are difficult teachers frequently provide materials at a considerable cost to themselves in time and money.

The equipment needed for play at this age must be large enough for the children to feel themselves actively taking part in the experience. Shops should be big enough for them to serve in.[1] Scales and weights, tape measures, liquid measures, etc., should be full size. They need also a good supply of cardboard coins, a large clock-face and real-looking goods to sell. Home play will demand actual cooking and washing materials; tables must be set for meals which will actually be eaten.

Experiences of this kind in school will differ from those in the home because they will be directed by the teacher. She will see that the children are put in the way of realizing the numerical aspect of their occupations. The use of specially devised apparatus will direct the children still further towards understanding. But it must be remembered that this is the first artificiality in connection with number teaching and requires considerable care and tact on the part of the teacher when she introduces it. It is assumed that class instruction will not generally be attempted but that the apparatus will be used by individual children as they are ready for it. Variety is of great importance if the apparatus is to help towards an understanding of number symbols and their generalized meaning. Bright colour and a connection with their natural interests and play will serve to attract the children. The use of this specially designed material is to serve as a stepping-stone from the use of natural material to number operations carried out with symbols alone. There will be a long period when it will need to be used alongside the symbols. It is suggested that work with apparatus should be planned not to teach the analysis of numbers, which is not desirable in the Infant stage, but to build up a knowledge of number combinations as a prelude to formal memorizing in the Junior School. It is urged that natural materials and occupations should not be discarded in favour of this didactic apparatus but should remain the more important element in number work for Infants.

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[1] It is assumed that this work will be taken in groups, the rest of the class being provided with other occupations. A shop to accommodate two children to act as shopkeeper and assistant can easily be made in folding form with a front and two hinged sides from light laths and hessian.

It is now recognized that, particularly in the Infant School, practical number experiences should be far in advance of any formal arithmetical work attempted; thus the five-year-old child will be able to count up to numbers he cannot write down; the six-year-old will undoubtedly be able to perform exercises in addition, subtraction and even multiplication with the help of apparatus and in connection with his interests but it is not desirable that formal work on these rules should be attempted at this age. The child entering the Junior School at the age of seven should have had wide quantitative experience gained from his daily life and from directed observation and play; he should have a vocabulary showing his familiarity with the simpler units of measurement of time, space and concrete quantities within his own experience; he should be able to perform such computations as he himself requires to do and to record his results, though not usually his methods of obtaining them. If no memorized results of calculation were expected the Infant School would be free to encourage much wider and more leisurely activities; a better foundation would be laid for later arithmetical knowledge and the child would enter the Junior School with a freshness of interest in number which would lead to easier learning at the later stage.

The arithmetical knowledge that may reasonably be expected of children at the age of seven is summarized in the paragraphs that follow.

COUNTING.

Counting forms the basis of all number work and it is natural that children should be able to count in a greater range of numbers than they can write down or use in computation. When the number names are spoken the difficult idea of place value is not involved; children can therefore learn the organization of large groups of objects into tens and hundreds, saying the numbers and thus preparing the way for writing them and for a later study of place value notation. The six-year-old child can count and write to 100 and will probably count beyond. If his counting is learnt through situations of real interest as in shopping activities or a party for parents, the large numbers will come easily to him and will hold his attention in a way that the abstract analysis of small numbers will fail to do.

NOTATION.

Though children differ widely in the extent of their knowledge of written numbers, some children knowing none at all on starting school, others being able to read four-figure numbers on bus tickets, it is important that all should have much practice in writing and reading numbers as they learn to recognize them. This is best arranged in connection with activities in which a large group or even the whole class co-operates. No formal teaching on place value is required. The tens will be recognized by their position and the majority of children will read hundreds in the same way but formal sums involving carrying are quite out of place at this age.

The symbols for the four operations +, —, ×, ÷, will be learnt later.

THE FOUR BASIC RULES.

As has been indicated earlier, children will learn *addition* easily in playing scoring games, buying and selling at the shop, and in replenishing its stock. By the age of seven they will be able to write down their transactions using the sign for adding. By constant practice they will come to know the results of adding pairs of numbers whose sum is 20 or less but memorizing these results deliberately is not advised nor is the working of examples set on paper without material of some kind to give them meaning. A few of the more advanced children will be able to dispense with apparatus but no attempt to force this should be made.

The method to be adopted in teaching *subtraction* is not a problem in the Infant School since formal work is not expected. What is needed at this period is that the children shall have experience of subtracting in its several forms, as finding a remainder when goods are sold or biscuits eaten, etc., as the number of things required to bring the stock of the shop up to the correct amount (complementary addition), as the number by which one group is bigger or smaller than another, and as the quantity sold from known stock when what is left is counted. If this foundation is laid any of the standard methods can be taught later without difficulty. By seven years of age the children should be able to write these various operations with the symbol for subtraction but should not be expected to work mechanical exercises upon them.

Multiplication as a set process is not suited to the Infant School since it implies that tables will have been memorized. It should be known to the children as counting groups and its sign can be used with that meaning. A child who has learnt to count in twos or threes and to write down the results of making such groups will be ready for learning tables after the age of seven and for formal multiplication. Experience in grouping will come readily in shop work, where goods are sold in sets, and in transport projects, since people sit in twos in buses and in fives (fours, etc.) in trains.

It is generally agreed that *division* is the most difficult of the four processes and should consequently be learnt some months or even a year later than the corresponding work in multiplication. It therefore seems wise not to attempt any interpretation of division beyond the simple ideas of sharing and grouping and to keep this closely related to real problems.

Units of Measurement.

If children develop interests in several kinds of shop, e.g. a grocer's, a draper's, and a milk bar, measurement of weight, length and capacity become necessary; they will want to tell the time; they will need to use money and give change. The extent of what they can learn in this way is set out below.

Coinage.

The children should learn to recognize the smaller coins but not the florin, the half-crown, the ten-shilling note or the pound note. They should know the exchange values up to a shilling and be able to give change for sums up to a shilling; they should be able to *count* money up to ten shillings.

Weight.

They should be familiar with weight as a property of things and as distinct from size. They should use a pair of scales with pound and ounce weights; ½-lb. and ¼-lb. weights are not recommended at this stage but children can weigh beyond 1 lb.

Length.

They can understand length as a physical property. They can use a foot rule including inches and a yard measure. The English measures on Montessori apparatus can be used for this.

Capacity.

They should know capacity as the amount a vessel contains, and the size of a pint and a quart. They can learn that three school bottles make one pint. (A name for this unit is needed in schools; *scottle* is a possible form.)

Time.

The children should have some knowledge of duration of time and have begun to notice the movements of the hands of a clock. They should recognize times that are important for them.

SPATIAL KNOWLEDGE.

Many Infant Schools now provide apparatus for children to develop their knowledge of shape and size. Young children play fitting games of all kinds and will spend long periods putting shapes into slots or through holes. Occupations of this kind not only bring understanding of spatial properties such as shape, size and direction but appear to give many children deep emotional satisfaction. They also provide training in correlated movements of the fingers and judgment of distance and size. Jigsaw puzzles, mosaic, pattern-making with tiles, etc., provide exercises on the flat; the fitting of blocks will carry the work into three dimensions. In the 6–7 year many of the constructions the children will wish to make, e.g. stalls for a Christmas sale, a street scene, a cinema, etc., will bring understanding of the spatial conditions involved if the teacher directs observation along those lines.

THE EXTENT OF THE NUMBER COURSE.

Teachers sometimes urge Infant children beyond the limits here suggested because demands are made upon them by the Junior School which is in turn urged on by the requirements of the selection test for Secondary Schools. These requirements are now being modified by many authorities.[1] It is urged that teachers in Infant Schools can best help forward the designing of soundly constructed tests by teaching their children only such arithmetic as they consider is acquired without strain and with a sense of satisfying mastery.

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[1] See Appendix II.

CHAPTER II

SPATIAL KNOWLEDGE

> "There is a general agreement among our witnesses that too much time is given to arithmetic in primary schools and a general regret that too little attention is given to the study of geometrical form." *Report of the Consultative Committee on the Primary School*, 1931, p. 178.

In a commercial age like the nineteenth century the mathematical skill most useful to children was numerical calculation. To-day its importance has decreased. Computation is often done by machines; standardized products and prices have lessened the need for individual reckoning; tabulated results are available for many of the financial affairs of the ordinary citizen. Yet the other great subject of mathematics, space and its properties, remains of fundamental importance. Interest in shapes is older than the Egyptian pyramids and as new as the latest example of modern architecture or the design of an aeroplane.

In Nursery and Infant Schools it has been recognized that young children have a natural interest in the shape not merely of significant objects but of simple geometrical forms. Children who are allowed to choose their play material show pleasure in manipulating geometrical shapes, in fitting them together, classifying them and comparing them. Much of this kind of material is now used in Infant Schools to train children to perceive differences in shape, size and position. At the other end of school life, in nearly all forms of post-primary education, attention is again given to geometry, either practical or theoretical. Three reasons are advanced for this: the subject has a practical value in many callings; it gives opportunities for work with the hands; it is a valuable means of education since it offers a mode of logical thinking and also presents us with constructional problems that cannot be solved unless they are fully understood.

Between the Infant School and a Post-primary School lies a period of three or four years during which the study of geometry is often entirely neglected. Boys sometimes take a course which is admittedly taught as an aid to handicrafts alone, and consists chiefly of measuring and the use of instruments. It contains little investigation of spatial properties and few mathematical ideas of interest or importance. In much of the constructional work taken in the Junior School little use is made of the opportunities afforded of discovering the shapes and sizes required for practical ends. Too often the teacher feels obliged to save time by giving the class shapes already cut out, or by letting it work to detailed instructions; the children then fail to work with alert minds or to reach an understanding of the processes involved. Yet observation of children's play during these years shows a developing interest in spatial relations as can be seen in

(*a*) games of the jigsaw type,
(*b*) the making of pleasing patterns,
(*c*) the construction of models of interesting mechanisms or everyday things such as paper aeroplanes and darts, houses, dolls' furniture, wireless sets, boats, etc.,
(*d*) a later interest in charts, plans and maps.

The basic ideas of geometry, viz. shape, symmetry, size and position (including orientation or lie), are as important as counting and serve as a means of mental development particularly stimulating to children who find difficulty in work with abstract number. They provide a pictorial foundation for logical thinking which expresses itself in action rather than in words or symbols. Work with shapes in cardboard or wood that can be handled should precede the study of figures drawn on paper since it gives a sound understanding that some children cannot reach through the more abstract method. Symmetry is especially important at this age when children have just achieved the power to distinguish unerringly between such forms as d, b; N, И. Finally a knowledge of shapes other than the rectangle makes possible a more extensive though still practical geometrical course in the Secondary School.

It is not possible, even if it were desirable, to set out a geometry syllabus suited to all Junior Schools or to all children in any one school. We suggest instead certain topics which are suited to each of the four possible years of a Junior School and which constitute a logical development of mathematical ideas. In the earlier stages individual apparatus is a necessity and much ingenuity is required to provide sufficient material for a large class. Further experiments are needed to discover how exercises that are profitable for individual children can best be adapted to groups of more than twenty children. It is hoped that as teachers become freed from the compulsion to spend all the time available for mathematics on an overloaded arithmetic syllabus they will find time to select from the following suggestions and work out for themselves exercises and projects which will involve the more important ideas and skills that can be fostered at this stage.

SUGGESTED TOPICS AND EXERCISES.

A. 7–8 *Year.*

1. Work with tiles or cardboard shapes including the square, equilateral triangle, the circle, and their halves. The ¼-circle gives variety and provides interesting comparisons with the square. Exercises would include:

(*a*) free pattern-making using a given number of shapes,[1]
(*b*) fitting certain tiles into trays of a given shape, e.g.

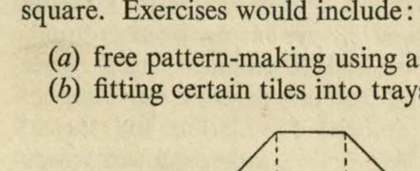

FIG. 1.
TRAYS REQUIRING THE SQUARE AND ITS HALF.

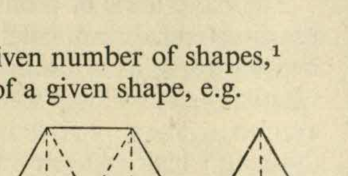

FIG. 2.
TRAYS REQUIRING THE EQUILATERAL TRIANGLE AND ITS HALF.

Several different arrangements are possible in some of these and lend increased interest and give understanding of the relations between the triangular shapes and those from which they are derived.[1]

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[1] Stencils cut in plywood or card are recommended for this.

(*c*) copying given patterns using at first the shapes indicated and later freely chosen shapes. Patterns for this age will often be realistic, e.g.

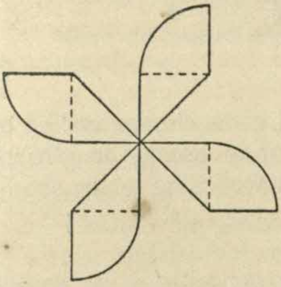

FIG. 3.
EASY EXERCISE.

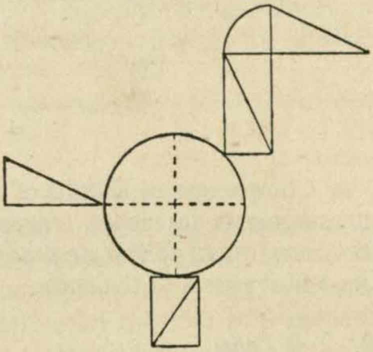

FIG. 4.
HARDER EXERCISE.

2. Paper-folding and cutting using square and rectangular shapes to make simple models such as an open box, a tower, chimney, chair, etc. Such work produces three-dimensional structures, an important addition which needs to be stressed since two-dimensional apparatus is usually easier to use under classroom conditions. Solid bodies, cubes, pyramids, spheres, etc., should also be handled by the children but it is more difficult to devise puzzles and exercises. The fitting of such solids into hollow cubes and open boxes is instructive; opportunities for building up pictured forms and for free imaginative work should also be allowed,[2] e.g. using blocks to make model villages, streets, aerodromes, etc.

3. Exercises on symmetry such as

(*a*) Paper-folding to make symmetrical shapes.

(*b*) Pricking through to obtain the mirror image of a given pattern; lines drawn through the pricks then complete the picture.

---

[1] Insets and their appropriate frames can be made by cutting out the shapes in plywood and making a backing of an equal sheet of plywood for the frames.

[2] A pin-and-hole device is recommended. If holes are drilled in all the faces of each block and wooden pegs provided a variety of forms is ensured.

(*c*) Putting in the missing parts in drawings of real symmetrical objects, e.g.

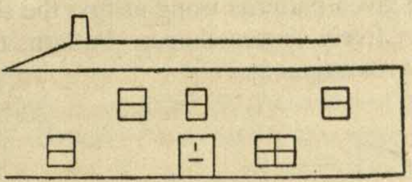

FIG. 5.

4. Comparison of lengths of sides of given shapes leading to measurements in inches. Measurement of greater lengths in feet could either follow or precede this according to the activities and interests of the class.

B. 8–9 *Year.*

1. More elaborate exercises with tiles and patterns, extending the shapes to include the general rectangle, its half made by a diagonal cut, triangles of various shapes, the one-sixth sector of a circle and the corresponding segment. The analysis of shapes into their component shapes.

2. Playground and indoor measurements connected with rectangles, including the lengths of their diagonals. Practical ways of making a square corner, e.g.

(*a*) through folding an irregular sheet of paper as shown in figure:

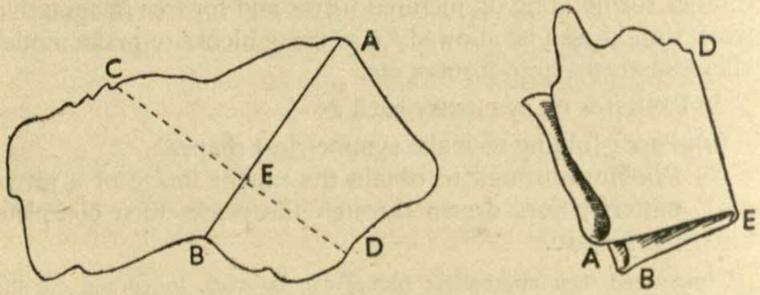

FIG. 6.

First fold—AB.
Second fold—CED

(*b*) through erecting a vertical stick using a plumb line.

Marking out rectangles in playground or hall; setting out North-South and East-West lines on the ground.

3. Making models involving the isosceles triangle using only the length of the sides and symmetry; e.g. envelopes, a house, pyramids. These can easily be constructed with squared paper, or through folding.

4. More advanced exercises on symmetry using additional geometric forms.

C. 9–10 *Year*.

1. Area of irregular rectangular shapes by counting unit squares leading to the rule for the rectangle and its half.

2. Finding the distance of an object from a wall by means of a rope and square corner leading to the idea of a perpendicular. Drawing simple plans to scale and fixing objects in them. Simple elevations. Drawing parallel lines using equal perpendicular distances.

3. Finding the compass points by folding a paper circle; simple traversing with a compass.

4. Extending the idea of an angle (hitherto thought of as the shape of a corner) by use of the clock-face. Exercises showing that change of shape involves change of angle as, for example, by deforming loosely hinged wooden or Meccano frameworks.

Copying angles using

(*a*) strips of paper on which parts of the arms are drawn,
(*b*) pricking through,
(*c*) cutting or drawing round.

Copying rectilinear shapes using these methods for the angles.

D. 10–11 *Year*.

1. The study of deformable shapes (hinged frameworks) leading to the idea of a tie-line.[1] Setting out and measuring quadrilaterals using a tie-line.

2. Drawing circles, at first with ropes, later with simple com-

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[1] Four Meccano rods bolted together to form a quadrilateral can be made rigid by inserting a diagonal rod as tie-line.

passes; finding hidden treasure when distances from two fixed points are given. Circle patterns with compasses.

3. Copying rectilinear shapes using an ungraduated protractor.[1] Making three-dimensional figures such as pyramids, prisms and the wedge. Making model castles, churches, huts, tunnels, wagons, trucks, furniture, etc., in connection with interests that may arise outside the arithmetic lesson. Further work in plans and elevations.

4. The study of parallel lines in trellis patterns; observation of angle relations informally. Finding hidden treasure given distances from lines.

It should perhaps be stressed that we do not advocate the insertion of another subject in the Junior School curriculum; we suggest that much of the work outlined above could be incorporated in the arithmetic scheme; some would be found more fittingly associated with geography; handwork for girls as well as for boys would give opportunity for the constructional parts; needlework lessons can sometimes be devoted to the making of decorative geometric patterns, since the stitch is still one of the simplest ways of making a straight line.

It will be noticed that in general the exercises do not call for great precision in the use of instruments. Children of this age-range still have difficulty in fine control; the proper place for training in the accurate manipulation of geometrical tools seems to us to be the post-primary school. For this reason measuring with a graduated protractor is not advised.

Formal geometry is not expected at any period of the Junior School. It will be seen in the first topic suggested for the fourth year that no reference is made to congruent triangles, yet the work with tie-lines enables the children to see that the triangle is the fundamental shape in engineering and constructional work because it alone of all rectilinear figures is rigid. We believe that a better foundation for later mathematical training is provided by practical experience of this kind and by the enjoyment derived by children from pattern making, than by the retention of some of the formal computation now taught.

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[1] A circular disc cut out in tracing paper and marked to show the centre and one radius can be used for comparing and copying angles.

CHAPTER III

THE FOUR RULES IN NUMBER

At the beginning of Junior School work the teacher's first task is to help the children to formulate and memorize the arithmetical ideas which they have met informally in an Infant School. The means of acquiring this informal number knowledge have been outlined in Chapter II. There is no doubt that a child's number sense and pleasure in dealing with numbers can be spoilt by expecting him to learn too early facts which are not part of his world, and by depriving him of the interest of number apparatus and number games. It is therefore important that the Junior School teacher should understand thoroughly the methods used in the Infant School and that apparatus similar to that previously used should be available during the first year in the Junior School and for more backward pupils throughout the course.

The number facts with which the children have become acquainted in an Infant School must now be gradually committed to memory. Charts and pictures may take the place of some of the individual apparatus. Constant practice should be given in the number combinations obtained by using the four fundamental processes. Children will then avoid laborious counting in ones and the repetition of a whole table to find a single item. This drill can be practised enjoyably either in small groups with the teacher or occasionally with a reliable child leader and with individual apparatus.

In this work reasoning about numbers is beginning to develop. The children's knowledge of number should be applied to experiences in the classroom and also to practical situations which are described to them so that work with abstract number is a generalization of these experiences. As processes are introduced they should in the first case be demonstrated with apparatus or with material in which children have a spontaneous interest. The children should at each step pass through the stages of experiment, association, practice and back to applications to real life situations.

The basis of the Junior School work is the thorough understanding and mastery of the addition and subtraction number combinations, the multiplication tables and the tens number system. Throughout the basic work sticks and elastic bands (or similar material) should be provided and used by the teacher for demonstration purposes and by the individual children who are found to need them.

THE ADDITION AND SUBTRACTION NUMBER COMBINATIONS.

All combinations up to a sum of 10 should be revised and learnt so that for instance

$$5 + 4 = 9, \qquad 9 - 4 = 5, \qquad 9 - 5 = 4$$

are known at a glance.

This is part of the work of an Infant School but the experience of the Committee is that many of the pupils at the beginning of a Junior School course are still "counting on their fingers." It is very important that these children should consolidate the early work.

The combinations to 12 can be practised in preparation for the work with units of length and money.

THE MULTIPLICATION TABLES.

Assuming that a child has already met with the first six items of the tables of twos, threes, fours, fives, and tens, he should, between seven and eight years of age, record them in a book of his own and realize how to continue his table by addition. But again backward pupils should not be neglected and it may be necessary to begin the tables from the beginning with apparatus.

"THE TABLE OF TWOS" AND THE "TWO TIMES TABLE," ETC.

In whatever order the figures are recorded at first, whether $2 \times 3 = 6$ or $3 \times 2 = 6$, it is advisable to teach the "table of twos" rather than the "two times table,"

e.g. one two is two,
two twos are four, etc.

By this method the number 2 predominates and each item may be found from the preceding item by the simple addition of 2.

As each table is built up it should be written in the children's record books and on a permanent chart to be kept in the classroom. These children's record books should be carefully checked.

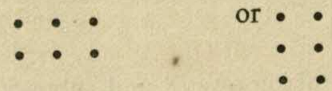
In addition to the separate table charts a multiple chart should be begun and systematically continued thus:

1	2	3	4	–	–	–	–
2	4	6	8	–	–	–	–
3	6	9	12	–	–	–	–

As the later tables are compiled, the children should be directed towards the discovery of interesting facts about the numbers and the way they are written, e.g. nines and elevens for the way the figures go, eights and twelves for their connection with fours, sixes, etc.

As soon as the tens table is known nines and elevens are easily compiled and learnt. Apparatus for these tables would be clumsy and should not be needed.

As the work progresses the children should learn all the relations of the form 2 × 3 = 3 × 2. This should be seen by the children as alternative groupings



The number of items in multiplication tables that need to be memorized is much smaller than teachers sometimes realize. If the children realize early the equivalence of 3 × 6 and 6 × 3, etc., the number of additional items in any new table is much reduced. Practice in the new table can then be concentrated on the items which are fresh. It is important to notice that this practice should not be entirely verbal. Some children learn by seeing rather than by saying and table practice should include a variety of ways, written as well as spoken.

Each item should be completely known and recognized at sight or sound when it is isolated from the table. To achieve this each table should be practised in haphazard order. Special attention should be given to the combinations which the children find difficult, usually those involving 0, 7, 8, 9.

THE TENS SYSTEM.

The children need to understand the notation of the tens system before they can learn addition and subtraction with numbers above ten. Apparatus (e.g. sticks and elastic bands) should be available during the early stages. The children should formulate their previous knowledge and extend it to include hundreds, thousands, etc. The values and notation of these larger numbers should be understood before they are used in addition and subtraction examples. Practice should be given in reading numbers.

ADDITION.

In the first stages of addition colouring the tens figures differently from the units figures may be found useful. The children will now be able to add to the extent of their knowledge of addition combinations, i.e. probably to 12. For harder numbers, e.g. 7 + 9, it may be found necessary to teach them to build up to the next ten, e.g. (7 + 9); 7 and 3 are 10; 10 and 6 are 16; but many teachers secure greater speed and accuracy by letting children memorize the additions up to 20. The method of addition need not be rigid and numbers may be grouped and facts used from the multiplication tables as the work progresses.

SUBTRACTION.

Subtraction by "taking away" and subtraction by complementary addition seem to be equally simple as first ideas of subtraction. When the children have advanced to the working of such examples as 23 — 17, there are three possibilities: subtraction by decomposition, by equal additions, or as complementary addition. Unless they are to learn the method chosen as a routine that gives the correct answer, it should be demonstrated with apparatus. The demonstrations of decomposition and complementary addition readily appeal to the understanding of a child, but that of equal additions seems more cumbersome.

On the other hand, at a later stage subtraction by equal additions is considered by some authorities to be simpler to remember and to give more accurate results.

At the early stage it is essential that whatever method is chosen it should be clearly explained and that the preliminary examples should be worked with apparatus and carefully related to the written examples.

There is one particular aspect of the method of complementary addition. It is the shopkeeper's method of calculating and giving change. Thus it would seem that subtraction of money would be better taught by complementary addition. On the other hand the sums of money involved in giving change are comparatively simple. The sum for which change is given is usually a "round sum." In some examples, the larger sum of money is frequently not a "round sum" and the working in this case often becomes complicated. Thus the "shopkeeper's method" plea cannot be entirely justified.

Whichever method of subtraction a school chooses to adopt, the teachers concerned should keep the following ideas in mind:

1. The method should not be inconsistent with any method which may have been used in the Infant School.[1]
2. The method should be carried on to the subtraction work in money, weights and measures, etc.
3. Difficulties may arise if schools in the same locality choose different methods.
4. Children joining the school after the early stages should not be expected to change their method.
5. A single universal method is not to be recommended.
6. Whichever method is chosen the "shopkeeper's method" of giving change should be explained and practised with suitable sums of money.

The difficulty of choice of method certainly points to the postponement of formal subtraction until the Junior School. It should seldom be taken in the Infant department.

MULTIPLICATION.

When the children have enough number knowledge multiplication seems usually to be done more accurately than subtraction. This is probably because by now the puzzles of place notation and of the carrying figure have been solved.

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[1] But see p. 5, paragraph 2, "in teaching subtraction . . . in the Infant School formal work is not expected."

The practice of short multiplication, using a table book for reference, is itself one of the most intensive methods of learning tables.

When a child has learnt that 3 × 6 and 6 × 3 are equivalent he will be able in such calculations as 867 × 5 to use the figures of the multiplicand as multipliers. Such an exercise can, therefore, give practice in a single table and also enable the child to perform operations which otherwise would be too difficult for him. In this case, for example, he will say "7 fives," etc., and it will not have been necessary for him previously to have learnt the table of sevens.

Division.

In an Infant School, division is learnt in a simple way as sharing and as grouping.

e.g. 10 ÷ 2 can be interpreted as follows:

(*a*) If 10 is shared among two people how many will there be in each share? Answer 5.

$$\boxed{1\ 1\ 1\ 1\ 1} \qquad \boxed{1\ 1\ 1\ 1\ 1}$$

(*b*) If 10 is made into groups of twos, how many groups will there be? Answer 5.

$$\boxed{1\ 1} \quad \boxed{1\ 1} \quad \boxed{1\ 1} \quad \boxed{1\ 1} \quad \boxed{1\ 1}$$

It is important to notice that

(*a*) belongs to the two times table,
(*b*) belongs to the table of twos.

The children now need to formulate these ideas and to learn to invert the items in the multiplication tables, e.g.

$$7 \times 6 = 42.$$

Thus the number of 6's in 42 = 7
„ „ „ 7's „ 42 = 6

By degrees division automatically becomes inverse multiplication, in which a missing term, either the multiplier or the group, has to be found, i.e.

$$\square \times 4 = 20, \text{ or } 5 \times \square = 20.$$

After this first stage in reversing the items in the tables practice in adding a number to a multiple may be given, e.g.

$$3 \times 5 + 2 = \boxed{}.$$

This leads on to division with a remainder.

Example. 17 ÷ 4. Practice can be given at first in fitting in 17 into the fours table, i.e. between 16 and 20; for this the multiple chart already given is useful.[1]

FORMAL SHORT DIVISION.

This is more easily understood as division by sharing.

e.g. 26 ÷ 2 Two tens arranged in 2 groups gives 1 ten in each group;
Six ones arranged in 2 groups gives 3 in each group.

Thus it follows that both methods of division, by sharing and by grouping, should be learnt.

In the setting down of formal short division the Committee suggests that from the start the answer should be placed *above* the dividend in the correct column.

Some children are helped in their first formal division sums by setting out all the working as in long division. Where they do this they should be encouraged to dispense with this help as soon as possible.

Examples in division need careful grading so that difficulties are met with one at a time.

Example 1. 16 ÷ 4 within the tables and using only one table.
" 2. 84 ÷ 4 beyond the twelfth item of the table but each digit divisible by 4.
" 3. 26 ÷ 4 within the tables but involving a remainder.
" 4. 128 ÷ 4 first two digits used as first dividend.
" 5. 264 ÷ 4 first dividend involving remainder.
" 6. 408 ÷ 4
" 7. 828 ÷ 4 } to introduce the difficulties of the 0's.
" 8. 243 ÷ 4

LONG MULTIPLICATION.

The children should discover the rule for multiplication by

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[1] See p. 17, paragraph 2.

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10 by observing what happens to the figures in given examples. Multiplication by small multiples of 10 may then be taught.

In passing to multiplication by 23 it seems more reasonable to multiply first by 20 then by 3; but the understanding of the method is more important than the order of multiplication.

LONG DIVISION.[1]

The obvious difficulty at the beginning of long division is that the table of multiples of the divisor is not known. If the children have at some time written out the working of a short division sum the long division pattern will be familiar.

If the children begin by making part of a table for a divisor such as 31, 41, etc., they will gradually learn how to estimate the figure of the quotient by comparison of the tens figures. At the first stage numbers such as 21, 31, 41, should be used as divisors till the method is grasped; then proceed to 32, 33, etc. After that 29, 39, can be regarded as approximately 30, 40, in order to find the quotient numbers. Finally the more difficult numbers 26, 37, etc., are reached and the carrying figure must be considered in making an estimate.

Applications should be restricted to examples of practical significance.

While practising these four rules sums may sometimes be checked by reversing the process and a fuller understanding gained. As skill in the four rules is the basis of all calculation, no child can feel mastery and pleasure in dealing with numbers or problems involving numbers until this skill is achieved.

SUGGESTED ORDER OF TEACHING THE "FOUR RULES."

7–8 Years.

Building of tables of 2's, 3's, 5's.
Later, tables of 4's, 6's, 10's.
Table of 12's informally with coins.
Practice such as 2 × 3 = 3 × 2.
Addition to 20 in table form.
to 100 in column form.

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[1] In this memorandum, the term "long division" is used to indicate division by numbers outside the tables.

Subtraction to 20 in table form.
to 100 in column form.
Short Multiplication within the known tables.
Division from the tables

$$3 \times 5 = 15 \therefore 15 \div 3 = 5$$
$$15 \div 5 = 3, \text{ etc.}$$

8–9 *Years.*

Tables of 12's linked with 6's.
Table of 11's.
Table of 8's linked with 4's.
Table of 9's linked with 3's and as (10 — 1).
Table of 7's.
Numbers extended to 1,000 and perhaps beyond.
Addition and Subtraction extended.
Short Multiplication within the tables.
Short Division within the tables as sharing.

9–10 *Years.*

Addition, Subtraction and Multiplication practised.
Long Multiplication begun.
Short Division extended in range and to include grouping.

10–11 *Years.*

Factors within the tables.
Easy long division.

CHAPTER IV

UNITS OF MEASUREMENT

The units of measurement may be arranged in two main groups:

Group I: Money.
Time.
Angles.

Group II: Length.
Weight.
Capacity.
Area.
Volume.

Group I.

The thing to be measured is abstract, but children are familiar with some of the units used: e.g. though money is a measure of value children accept it as a part of everyday life and as the means by which we buy and sell. Similarly time is intangible and yet children take for granted the time labels they hear used.

Group II.

Here there are two essential ideas and throughout the teaching the children should be constantly aware of the distinction between them.

They are:

(1) The physical property to be measured.
(2) The measurer or unit that is used.

A thorough consideration of the first should precede the teaching of the second.

(1) *The Physical Property to be Measured.*

Weight, for example, is a universal and absolute property of material things. It can be recognized by very small children and can be discussed in a simple way with them. They become aware of it long before they have any experience of the machinery used to measure it.

(2) *The Unit or Measurer.*

This is an artificial standard adopted by a group of people. It is different for different sets of people. It is not a property of the physical world. Thus it is essential that the children should become deliberately aware of the first as a natural characteristic and should learn the second as an arbitrary convention.

Before the age of seven children become acquainted with the simpler coins and with the idea of measuring some of the physical properties of things. In suggesting a syllabus for teaching the units of measurement for a Junior School it is, therefore, necessary to know what may have been learnt before entry into a Junior School. (See Chapter I.)

Despite some inconveniences in usage, our English system of measurement is of great interest since it is based on the old historical and traditional units. A historical survey of measurement is interesting and provides a teacher with a natural approach to the introduction of units. Most of the units of length are derived from comparison with personal measurements and quite young children will be interested in the use of their arms, hands and thumbs as rough measures.

The knowledge gained in earlier experiences has to be formulated and extended in a Junior School. The relations between units should be set out in a table and memorized. Each child should keep his own record book and enter the tables as they are fully introduced. These reference books should be carefully checked.

At first the reference book will be used while working examples but always with the intention that the tables should be memorized.

Time should be given for learning the tables and a varied scheme of mental arithmetic planned to assist this.

MONEY.

This is one of the more difficult parts of the work because the English system of coinage is complicated. The working of examples on paper is of little value unless the work is the outcome of practical ideas of the value of coins and the way in which they are used.

It is possible that some children at the age of seven will not easily recognize all coins. They should be shown the real coins as they are learnt and many of them may need practical work with imitation coins. Throughout the work in the Junior School imitation coins and notes should be available for individuals who are found to need them. Their use will help considerably in clearing difficulties which may not be completely overcome by verbal explanations.

Current price lists of all kinds, menu cards, etc., may be supplied for some individuals or for members of the whole class. It is important that the children should learn to read these lists, cards, etc., and to interpret them themselves and that problems and bills should be made from their suggestions and not merely given to them by the teacher.

Method and Arrangement of Work.

7–8 *Year.*

The work in the first year of the Junior School should consist of learning the coins by using them in a variety of ways such as shopping games and by working simple written examples.

There are three main ideas to be considered by the teacher:

(1) The method of addition and subtraction to be used.
(2) Source and types of examples.
(3) Practical work with coins.

(1) *Methods.*

The children should learn the analysis of 12, i.e.

$$\begin{aligned} 1 + 11 &= 12 \\ 2 + 10 &= 12 \\ 3 + 9 &= 12 \quad \text{etc.} \end{aligned}$$

so that the conversion of pence to shillings may be made directly by building up to 12 and that addition and subtraction of pence and shillings may be clearly understood from the beginning as a twelves system not to be confused with the tens system of numbers.

Methods of carrying in Addition.

Example 1.

$$\begin{array}{rl} \text{d.} & \\ 7\frac{1}{2} & \quad \frac{1}{2}\text{d.} + \frac{3}{4}\text{d.} = 1\frac{1}{4}\text{d.} \\ +\ 2\frac{3}{4} & \quad \text{i.e. } \frac{3}{4}\text{d. needs } \frac{1}{4}\text{d. to make 1d.} \\ \hline 10\frac{1}{4} & \quad 1\text{d.} + \frac{1}{4}\text{d.} = 1\frac{1}{4}\text{d.} \\ \hline \end{array}$$

It may be found necessary for some children to make a separate column for the farthings. This should not be insisted on for the children who can work the sum in the given form.

Example 2.

$$\begin{array}{rr} \text{s.} & \text{d.} \\ 2 & 7 \\ +1 & 9 \\ \hline 4 & 4 \\ \hline \end{array} \qquad \begin{aligned} 9\text{d.} + 7\text{d.} &= 12\text{d.} + 4\text{d.} \\ &= 1\text{s.} + 4\text{d.} \end{aligned}$$

If the method of 9d. + 7d. = 16d. is used there is always the confusion of carrying one ten instead of one twelve.

Subtraction.

Comment on the methods of subtraction has been made in Chapter III.

The two following examples illustrate the shopkeeper's method of giving change. If the method of subtraction adopted is by decomposition or by equal additions then the shopkeeper's method should be given as a special exercise, related to shopping play in the classroom.

Example 1.

To find the change from 1s. when spending 7½d.

7½d. needs ½d. to make 8d.
8d. needs 4d. to make 1s.
Answer 4½d.

Example 2.

To find the change from 5s. when spending 2s. 9d.

2s. 9d. needs 3d. to make 3s.
3s. needs 2s. to make 5s.
Answer 2s. 3d.

(2) *Types of Examples.*

In this first year the addition examples should not have more than four items nor the total sum be more than ten shillings.

Every effort should be made to give examples on practical school and home problems, e.g.

Cost of class milk supply,
Cost of class outings,
Savings,

Simple bills made from menu cards, simple current price lists, and charts of priced articles from advertisements.

(3) *Practical Work and Apparatus.*

(*a*) A plentiful supply of coins should be available. Group work and individual work should be organized for children as they need it, especially the backward children.

(*b*) Priced illustrations from newspapers, periodicals, catalogues, etc., may be put on to large charts. These may sometimes take the place of the classroom shop and may be used with groups or with the whole class.

8–9 *Year.*

Knowledge of coins and notes should be completed in this year.

The addition and subtraction work should be extended to include pounds. Sums should not involve quantities greater than £10.

The shillings column should be built up to pounds through the ten-shilling note.

Example.

£	s.	d.	
2	15	0	9 + 5s. = 14s. = 1 ten-shilling note + 4s.
4	9	0	2 ten-shilling notes = 1 pound.
7	4	0	*Answer*

After this has been learnt the children can work examples involving £ s. d.

Short Multiplication.

Before this is taught conversion from pence to shillings should be learnt as an application of the table of twelves.

Only easy examples should be given.

Change of Units.

Easy examples in the changing of units for sums less than £1 may be introduced. Long conversion examples and those which have no practical significance should be excluded.[1]

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[1] "Conversion" is thought to be a better description of the process of changing from one unit to another than the word "Reduction."

9–10 *Year.*

Harder examples in addition, subtraction, multiplication should be introduced here, extending the work to larger sums of money involving quantities above £10.

Short Division.

This should be taught as SHARING, and only simple examples should be attempted.

The examples may be set down long division way when "carrying" is first met.

Example.	s.	d.	
	1	11	*Answer*
	3)5	9	÷ 3
	3	24	
	2	33	

Change of Units.

Conversion may now include changing from pounds to shillings, and vice versa, within the limits of reasonable sums of money.

10–11 *Year.*

Harder examples may be practised in the four rules; in particular, simple problems involving two-shilling piece and half-crown, and the simpler fractional parts of 1s. and £1.

Short Division.

Short division as measuring, or counting groups, may now be taught, provided that the amounts are simple.

Example.	s.	d.	
	6	8	÷ 4d.
	12	72	
	72	4)80	
		20	*Answer*

LENGTH.

7–8 *Year*.

The idea of length as a distance should be discussed, both curved and straight distances being included in the discussion. The children's previous knowledge should be discovered and formulated; this will, at this stage, be vague, and by working for a time without a constant unit the children can be brought to feel the need for a greater precision. It will then be possible to introduce the necessity of working with an agreed standard unit. For measuring in the unit a suitable ruler should be used. Fig. 7 shows the type of rule found most convenient for this age.

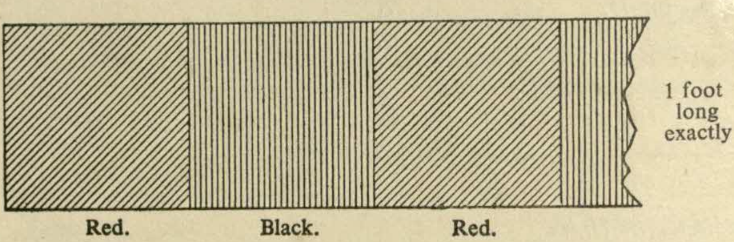

FIG. 7.

One side blocked in inches coloured alternately.
Reverse side plain and ungraduated.
No bevelled edges. *No* figures on the inch lines.

In the first instance children should make their own rulers from strips of card or paper 1 foot long. They may be given strips over a foot long and cut them down to the foot length using the ruler. Inch-squared paper may be provided for the marking off in inches—each child being given one inch square. The *making* of this ruler is valuable in itself as a practical exercise for learning the size of the units.

Examples.

Lengths should be estimated by observation,
(1) in feet to the nearest foot,
(2) in inches to the nearest inch,
(3) in feet and inches.

The children should write their estimates down and then check them carefully by measurement using their own card rulers and working individually or in groups according to the type of example.

This work with a simply scaled ruler familiarizes them with the units in a way that a ruler with fractional graduations will not. Yard measures marked only in feet may be used in the same way.

Written examples.

In simple addition the analysis of 11 and 12 will be used and the inches added in the same way as pence are added, i.e. building up to one foot.

The method of subtraction used should be consistent with the method used for number and money. For these examples no more than two units should be introduced at a time.

Carrying may be done, but should be limited to 1 foot or 1 yard only.

8–9 *Year.*

Harder examples of addition and subtraction using yards, feet and inches together should be worked.

The Mile. This should be discussed in a practical way, e.g. public notices on signposts, milestones, speed limit signs, speed records in newspapers.

The distance between two well-known local landmarks one mile apart will illustrate the length of a mile.

9–10 *Year.*

The sub-divisions of a mile should be taught. The following information will need to be told:

(1) 1,760 yards = 1 mile,
(2) 1 chain = 22 yards = length of a cricket pitch,
(3) 1 furlong = 10 chains,
(4) 1 furlong = $\frac{1}{8}$ mile = 220 yards.

The chain measure may be used in simple outdoor work where this is possible and where children can be organized in small groups.

Addition and subtraction may be extended to include these new measures but only in connection with practical situations. Short multiplication and short division (as sharing) may now be taught for yards, feet and inches. Simple exercises in drawing to scale should be introduced; lengths and ratio should be simple, e.g. 1 in. to 1 yd.; ½ in. to 1 ft.

The fractional side of the ruler may be used to illustrate the vulgar fraction work done during this year. Fig. 8 shows a suitable form of ruler.

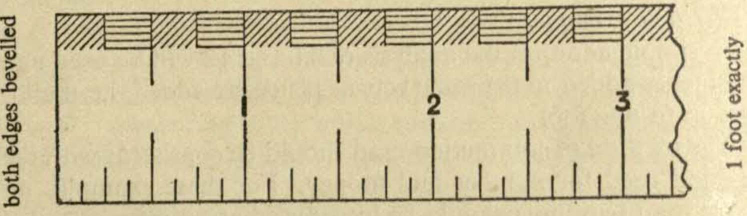

FIG. 8.

One side as diagram. *Top edge* blocked in contrasting colours for *quarter* inches; half-inch and inch *lines* in a third colour.

Lower edge lined only as diagram.
Reverse side as for 7–8 year: inches coloured in red and black.

10–11 *Year.*

Harder examples in work on the four rules and short division as measuring should be introduced. Simple drawing to scale and the making of plans should be continued. The work on areas has been introduced (see pages 13 and 36) and examples involving length and area should make a clear distinction between them.

Any special lengths used in the locality should be demonstrated (e.g. the pole in some rural districts).

Some simple work on the conversion of yards, feet and inches may be practised within the limits of practical usefulness.

WEIGHT.

7–8 *Year.*

The idea of the physical property of weight should be discussed and a record of observations made before units of weight are introduced.

The children should note everyday things which they are accustomed to carry which are heavy or light. Articles should be lifted and the children should estimate which of two things is the heavier. After this a pair of scales should be provided and there should be a practical discussion on machines for testing weight. The workings of an ordinary beam balance should be investigated and experiments in comparing the weights of various articles should be carried out to demonstrate the following possibilities. (No standard weights need be used.)

(1) Same size and appearance giving different weight (e.g. two similar boxes filled with different materials feel different when held one in each hand and do not balance the scales when they are placed one on each side).
(2) Larger size giving lighter weight.
(3) Larger size giving heavier weight.
(4) Same weight but different size.

The comparisons should be made by the children feeling the articles and then testing them on the scales.

Weighing out 1 lb., 1 oz. and small numbers of pounds or ounces of such material as sand, beads, etc., can be planned in connection with the classroom shop.

This should be followed by using the weights 1 oz. and 1 lb., ¼ lb., ½ lb. for finding the weights of various objects and materials. If necessary the fractions ½ and ¼ should be carefully explained but the ideas should be familiar from the knowledge of ¼d. and ½d. The children should themselves do much weighing of this kind. The abbreviations oz. and lb. should be carefully taught.

The Weights Table.

That 16 oz. make 1 lb. may be discovered by balancing 4 separately weighed ounces against ¼ lb. weight, and by actually balancing 1 lb. against 16 separate ounces. Then

4 times 4 ounces makes 1 pound,
i.e. 16 ounces make 1 pound.

Examples.

Simple addition and subtraction may be taught, in pounds and ounces, but no formal carrying.

8–9 *Year.*

Here further parts of the table may be introduced.

A 14 lb. weight (= 1 stone) or its equivalent should be in the classroom and the children's own weights may be discussed. (If the school weighing machine is not in metric weights it should be used.) Wherever the stone weight is used for household shopping the children should be familiarized with the local use.

Tons, hundredweights and quarters should be introduced.

20 cwts. = 1 ton.
a quarter = ¼ of a hundredweight.

Reference should be made to the use of the weights with coal. A sack of coal may be of various weights, usually expressed in terms of "hundredweight" units, e.g. ½ cwt., 1 cwt., 2 cwt., etc.

Notices on bridges and lorries may be referred to in connection with tons.

Easy addition and subtraction may be taught with two units at a time and no carrying.

9–10 *Year.*

112 lb. = 1 cwt. should be taught.
Then, 28 lb. = 1 quarter (involving short division).
and 2,240 lb. = 1 ton (involving × by 20).

Short multiplication and short division (as sharing) may be taught with pounds and ounces only. Addition and subtraction should be extended and practised.

10–11 *Year.*

Short division as measuring may now be taught in a practical way.

Further harder examples in the four rules but with not more than three units at a time should be given.

Simple exercises in change of units may be worked where possible with some practical application.

Capacity.

8–9 *Year.*

Previous knowledge needs to be recalled, and practical work should be done by measuring water in milk bottles and standard measures. ⅓ pint (school milk bottle), ½ pint, 1 pint, 1 quart bottles and a gallon measure should be provided.

The measuring of milk, oil, vinegar, petrol, etc., may be discussed.

The children should all have practice in finding the relative values and should enter the table in their reference books.

3 school milk bottles make 1 pint.
2 half-pints make 1 pint.
2 pints make 1 quart.
4 quarts make 1 gallon.

The abbreviations should be told.

There should be some discussion of the use that is made of these measures for dry materials, especially of the way in which they are used locally.

Similarly local use should determine whether children are taught about gills.

4 gills make 1 pint (the cookery book measure).

2 gills make 1 pint (the milk measure in some localities).

Easy addition and subtraction (with simple carrying) may be introduced, not more than two items at a time.

9–10 *Year.*

Easy short multiplication and division (as sharing).

Further practice in addition and subtraction, with three items.

10–11 *Year.*

Harder examples, including change of units and division as measuring.

TIME.

7–8 *Year.*

After a short discussion to discover what ideas the children have of the duration of time, length of intervals of time, etc., the clock-face and the movement of the hands should be studied carefully and the children should learn first to tell the time in hours, half-hours and quarter-hours and then to the *nearest five minutes past and to the hour*. This may be done by using a clock-face with removable numbers in *Arabic* numerals.

The children may need to learn the meaning of $\frac{1}{4}$, $\frac{1}{2}$ (as in money and weight at this age).

The number of minutes in an hour should be learnt.

The children may make their own clock-faces in paper (folding for $\frac{1}{2}$ and $\frac{1}{4}$). These may be stuck on to cards and hands added.

The main work in this year is to learn to tell the time.

They should also learn the length of the 24-hour day, the number of days in a week and the names of the months.

8–9 *Year.*

Telling the time in minutes (linked with table of 5's).
Rhyme for days in the month.
Number of days in the year.
Meaning of A.M. and P.M.

9–10 *Year.*

Telling the time in railway fashion.
Seconds.
Simple addition and subtraction in hours and minutes.
Examples.
Duration of journeys worked from current time tables.
Examples connected with transport and Post Office mails.

10–11 *Year.*

Simple problems on speeds and rates.

AREA.

9–10 *Year.*

Every child has an idea of what is meant by surface, but as a rule he does not separate the familiar property of surface until it is pointed out to him. Curved surfaces and plane surfaces need to be discussed with him. The possibility of covering a surface distinguishes surface from boundary line, e.g. in painting, ironing, scrubbing, dusting, etc.

Units of Measurement.

In the introduction of these units of measurement many difficulties arise. The unit is square in shape because that has been found to be convenient, but areas can be covered by a variety of unit shapes, e.g. by equiangular triangles, by oblongs, by regular hexagons, but not by circles.

The measuring unit is less easily manipulated than the pint or the foot because it is not readily adaptable to irregular shapes and surfaces.

Since the unit of area measurement is a square it may seem impossible that an area can be accurately found unless it is rectangular. In the elementary work this sometimes gives rise to the misconception that if a shape is not rectangular it has no area.

1 square inch is the amount of surface covered by a square of 1 inch side.
1 square foot is the amount of surface covered by a square of 1 foot side.
1 square yard is the amount of surface covered by a square of 1 yard side.

These units should be demonstrated with squares of the actual size and a rough estimate of some areas should be made by placing the suitable measure successively over the surface.

One of the measures may be torn into two or more pieces and the pieces placed so as to form an irregular shape, but still to cover 1 square inch of area.

As the units of measurement are squares, the areas of rectangular surfaces are the easiest to find.

The children should see an accurate representation of the facts that

144 sq. ins. make 1 sq. ft.
9 sq. ft. make 1 sq. yd.

Illustrative diagrams should be on view throughout the work.

10–11 *Year*.

The acre and square mile should be introduced and connected with some local areas.

Examples may be worked involving drawing to scale and showing a clear distinction between length and area.

In describing a rectangle whose size is 6″ by 3″, the notation 6″ × 3″ should be avoided since it leads to many misconceptions regarding area.

Some simple examples concerning surfaces of solid objects and consistent with the spatial knowledge at this age could be worked.

ANGULAR MEASUREMENT.

9–10 *Year.*

Angular measurement should be introduced as measurement of change of direction or of rotation. The children may first make diagrams of compass points by paper folding. A compass should be used to find North and the orientation of the classroom should be noted.

Practical class or group work should be done on a table or on a blackboard placed horizontally. A movable pointer should be used on a large diagram to show change of direction from one compass point to another, the results for clockwise and anti-clockwise directions being noted in terms of right angles and simple fractions of a right angle.

CHAPTER V

FURTHER CONTENTS OF ARITHMETIC SYLLABUS

This Report, which is primarily concerned with a minimum syllabus for children not above the average in intelligence, is based on the assumption that arithmetic is a utilitarian subject for all but the mathematical. To be able to apply arithmetic to daily needs is considered to be enough for the majority of children in Primary Schools and, to this end, it is recommended that the applications of arithmetic and its informative aspect should be stressed rather than mere computation. The calculations which are considered necessary should be taught so that they are precisely understood and can be speedily performed; they should, in all cases, be confined to reasonable figures. Scrupulous accuracy should be demanded within these limits.

On a foundation of accurate mastery of small and reasonable figures and of the ready application of arithmetic, the Secondary School teacher can build further arithmetical and mathematical work very much more quickly and through a more coherent scheme than is possible if more advanced work is attempted at the Primary School stage. Those to whom arithmetic has no interest outside its use in everyday life do not need, and do not reap any benefit from attempting, the more advanced work.

In our Primary Schools, the needs of two groups of children must be considered, (*a*) a majority who will require arithmetic chiefly, or indeed solely, for its application to daily life, and (*b*) a minority who will go on to the Grammar Schools and who, therefore, in addition, require arithmetic as a basis for further mathematical work. The needs of the majority must not be overlooked in catering for the minority who will go on to the Grammar School. It is probable that a simpler and more practical training will be more valuable to the latter also, than would be the type of work which has, on the whole, been attempted in the past.

In what follows consideration is given to those aspects of the arithmetic syllabus not included in the previous chapters.

VULGAR FRACTIONS.

In the Junior School fractional work should not be abstract. Fractions of real things should be dealt with, the manipulation of fractions of numbers being reduced to a minimum. The introduction of brackets in fractional work should be omitted and ladder fractions eliminated. Only such manipulative skill is necessary as will enable the children to deal with fractions of money, material, groups of objects, etc. Equivalence of fractions and then addition and subtraction through equivalence should be taught. Multiplication and division of fractions should be taught first by integers, then by fractions. In teaching division of fractions the first approach should be through division as sharing, followed at a later period by division as measuring.

The equivalence of fractions should be taught only informally up to the third year, but, after this, a little formal work on equivalence and on the manipulation of simple fractions of concrete quantities will be necessary. Addition and subtraction of fractions with different denominators should be limited to common denominators not exceeding 24. Little difficulty should be met with if, in the third year, the children have satisfactorily learnt to add and subtract fractions with the same denominators.

There seems little to commend the teaching of a formal method of finding the *lowest* common multiple. When the principle of a common multiple has been understood, the possibility of simplifying work with more difficult multiple numbers may be pointed out in dealing with a specific example.

DECIMAL FRACTIONS.

There exists some diversity of opinion amongst those who are intimately concerned with the Junior Schools as to the most satisfactory introduction to decimal fractions and the purpose underlying their inclusion in a Junior School syllabus.

In the opinion of the committee a normal child should, on leaving the Primary School, *understand* the decimal system of notation beyond the first place, but should not be required to go further than the first place of decimals in formal work. Some of the abler children will readily use the second place of decimals. Its connection with percentage, though important, should probably be kept as a topic for the Secondary stage. It is suggested that the interpretation of 98·4° temperature, the use of averages in connection with mark lists, or selected lists from newspapers could provide a real introduction to the subject, the child being led to appreciate that this method of expressing fractions is in use in everyday life and is, therefore, useful to him.

An alternative method of approach follows as an extension of the tens-notation. This is favoured by those who feel that decimals should be introduced formally and the discussion of their uses deferred until the child has some understanding of their significance. If an elementary analysis of the ordinary tens-notation is undertaken at a time when the children are old enough to understand the place values of digits in the tens, hundreds, columns, then the notation may be extended beyond the units figure to tenths, hundredths, etc., and decimal fractions be thus introduced formally.

In either case the committee is certainly of the opinion that early decimal work is confused by reference to the metric system, since, at the time when children are being introduced to decimal notation, they are unlikely to be completely familiar with the metric system. The introduction of the metric system is referred to later in this section.

The important thing is the understanding of the decimal notation to one place. This may be given to the children through its daily use, as suggested above, through temperatures, mark sheets, mileages on a cyclometer or speedometer fitting of a car, etc. Some exercises on tenths of an inch in measuring are valuable for consolidation work, but the child's limited accuracy in ruler work must be remembered.

Addition and subtraction offer no difficulties. They reinforce understanding of the tens-notation.

Short multiplication of decimal fractions and their division by whole numbers which are exact divisors may be attempted confidently. The extension of division to other cases should be limited to examples in which there is a remainder in the first place of decimals only.

The inaccurate phraseology often used in connection with the manipulation of decimals leads to confusion. Multiplication of a number such as 32·45 by ten gives the number 324·5, the place value of each digit being different in the two numbers. The description of this process either as "moving the decimal point" or as "keeping the decimal point fixed and moving the figures along" frequently obscures the fact that one is dealing with two different numbers which happen to be related to one another in a particular way. The convenience of the expression "move the point one place to the left" is a temptation, but too facile a use of it often leaves the child unaware that an actual multiplication is being performed. Such expressions should be used only after the children's understanding is assured.

The metric system may be introduced in the last year of the Primary School as an alternative to the British system of weights and measures. The metric system is an alternative and independent system of units and should be taught as such. The confusion created by unnecessary problems on conversion from one system of units to another is aggravated by the rulers in use in many Primary Schools. These rulers usually are graduated along one edge in inches and fractions of an inch and on the other in decimetres and an odd number of centimetres on the other edge, whilst the back is taken up by an inaccurate and unusable protractor. It would be far less confusing if the rulers provided for general use were graduated only in inches and fractions of an inch, while a set of completely metric rulers were available for use when the metric system is being used in class.[1] It is essential that the conceptions of a centimetre and a kilogramme be obtained directly and not through equivalence to inches and pounds. Conversion from one system to another follows much later and is not within the scope of a minimum course for the majority of Primary School children.

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[1] See Chapter IV, p. 32.

PROCESSES BEYOND THE FOUR RULES.

Unitary Method.

Unitary method is useful only in simple cases connected with shopping; it is limited and not of general application.

Ratio and Proportion.

The notion of ratio may be reached through the drawing of plans, scale drawings of the classroom or playground and through the interpretation and use of such plans as from time to time appear in the press. These may often be used for informal numerical work. A formal definition of ratio should be deferred to the Secondary stage.

Factors and Common Multiples.

During the final year of the Primary School some elementary analysis of numbers may be introduced such, for example, as the splitting up of a number into different pairs of factors. For this purpose it is considered advisable to restrict the numbers to those which are met with in the ordinary multiplication tables. When, for instance, it has been appreciated that 2, 3, 4, 6, 8 are all factors of 24 the subject of common multiples may readily be introduced. There is no point in insisting on a lowest common multiple and in fraction work it is not recommended that the method of finding the lowest common multiple be taught; yet it is felt that children should have it pointed out to them that by choosing a smaller common multiple they can make their work in some cases considerably easier.

The highest common factor is rarely of any use and no formal work with it should be attempted.

Graphs and Tabulations.

The ability to read simple graphs, such as classroom temperature graphs, mark records or any others which arise naturally and simply, is useful. In the same way tabulation for the sake of clarity is worthy of some attention, omnibus or railway time and fare tables being useful for this purpose. In neither case need Primary School children be asked to construct such graphs or tables, their interpretation being all that the children will require for some years.

SHORT METHODS.

"Short methods" should be approached only very simply and applied to such problems as the costs of dozens, scores, etc. No artificial short methods should be attempted and, above all, nothing introduced which will cut across the simple, known methods which have been carefully taught. These short cuts are very confusing to children who are just building up a confident mastery of arithmetic and in lightning calculation are a source of much of the discouragement and distaste which inhibit successful work at later stages.

TOPICS TO WHICH AN APPROACH IS POSSIBLE IN THE PRIMARY SCHOOL.

There is a danger that in our attempt to keep arithmetic in touch with daily needs and ordinary life, we may attempt to introduce subjects which are not truly of interest to the children of the particular age with which we are dealing. Much of our so-called practical work and real-life work in arithmetic may be criticized as being as fundamentally artificial to the children as were the old-fashioned text-book "sums." Some of our projects and topics involve arithmetical applications which are not intrinsic and are of neither real value nor interest.

We may use household bills and introduce children to current prices and usual quantities, but even here there is sometimes danger of artificiality. The Post Office affords a few topics of genuine interest to children; the purchase of postage and savings stamps and the use of the telephone are usually acceptable topics.

The newspaper provides some examples if careful selection of price lists and appropriate choice of advertisements are made. Records of rainfall and temperatures quoted in weather reports may also be used if they are taken very simply and if correlation with geography and science is possible.

Junior School children are interested in times and speeds in connection with vehicles, journeys and sports. Sports also furnish some examples of scores and averages.

The costs of various commodities may be introduced through topics and projects centred in the work of a baker or grocer, but care must be taken lest this too becomes artificial and unreal.

When gardening, either at school or in the neighbourhood, occurs as a real occupation, some simple work connected with it may be attempted.

In the view of the committee the present needs are the simplification of the arithmetical work of the Primary School, and a greater relevance to the interests and experiences of the children. The tendency to make the work artificial leads to the spending of too much of the children's time in ways that yield them neither interest nor benefit. We do not believe that a diminution of the time and attention devoted to arithmetic of a heavy, mechanical and abstract type at this stage will result, in the long run, in poorer work in the Secondary Schools, but rather that a firmer foundation of simpler and properly understood work will be laid as a sound basis on which to build the later work of all children.

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CHAPTER VI

METHODS OF TEACHING

INTRODUCTION OF NEW PROCESSES.

New processes and notations should be introduced, not in an abstract way, but, wherever possible, through realistic situations. These may be actual experiences in the school, recalled situations or hypothetical cases acceptable to the child's sense of reality. For example, an actual experience may be the weighing of an object in ounces and pounds in the classroom; a recalled experience may be that of standing on a weighing machine and reading the weight in stones and pounds, whilst the hypothetical case of imagining the job of a coalman could introduce tons and hundredweights. A wide range of similar problems should be considered, so simple that the pupils *discover* the new process through them, with a minimum of verbal explanation.

The standard processes of the four rules in number should become automatic mental habits; we see no need to expect the average pupil to be able to give a reason for the steps in every process. In fact whether the pupil be taught by the deductive method in which every step is seen to be a necessary part of a logical sequence, or by the inductive method, that is, by learning the process as a rule which is found to work, he eventually carries out the standard process by rule of thumb. There may, however, be a difference between the attitude of pupils taught by the two methods. There is a tendency for pupils taught by the inductive method to regard arithmetic as a mysterious ritual, the sequence of which must be memorized, whereas those taught by the deductive method who have some understanding of the reasoning feel that the development is more natural and less arbitrary. We, therefore, feel that the deductive method should, in general, be used in an introductory lesson on a new process in order that the process may be made *acceptable* to the pupil, but no more than an *appreciation* of the reasoning is looked for, and attention is quickly turned to the formation of habits. At this stage it would be absurd to cast every sum in problem form.

It is a common experience that some children appear to master the first steps of a new process, but fail to achieve general command of a new skill. This is usually an indication that the process has not been analysed into the many short steps that children of no more than average capacity require. The ability to make this analysis into easy steps is one of the most valuable qualities of a good teacher of Arithmetic.

DRILL WORK.

Once a process has been introduced through a practical problem in such a way that the pupil feels the need of achieving mastery of the process, it is necessary to concentrate on development of speed and accuracy in carrying out the process, but, in accordance with our general principles, we think that the skill in manipulation should not be developed at any time beyond the need of the practical problems which are within the range of the pupil's experience and interests at the time. The examples should be carefully graded and there should be enough easy problems to give the children a feeling of confidence. The drill work should be broken up much more than is customary by the application of the process to the solution of practical problems. A return to a practical situation at successive levels of difficulty in the new skill is one of the best ways of securing that the children understand what they are about.

DISTRIBUTION OF ORAL AND WRITTEN WORK.

We believe that there should be an increase in the amount of time devoted to oral work. As the most economical and efficient method of revising mathematics, oral work is commonly used, but insufficient use is made of it in helping the pupil to acquire skill and confidence in new processes.

It needs to be emphasized that arithmetic has its own language and that its vocabulary is best acquired orally. More use could be made of oral questions requiring written answers.

The practice of allocating certain periods of the week to "mental arithmetic" lessons does not commend itself to the committee because it is felt that there can be no clear-cut division of teaching into mental and written lessons. In practice these mental arithmetic lessons frequently are out of touch with the development of the subject in the other lessons and seek to achieve dexterity in using a fundamental process in isolation, carrying quickness in number work very far beyond the stage required by the children's ability and interests in practical problems. It is felt that the time could more profitably be spent if these lessons were an integral part of the systematic development of the subject.

PROBLEM WORK.

Questions in applied arithmetic are not necessarily problems. In many questions the process to be adopted is readily recognizable; the essence of a *problem* is that the process necessary for the solution is not obvious. We should like to see more problem method in the Junior School. Interpretation of a written problem is an excellent exercise; difficulties that occur in this direction can usually be removed by illustrating the problem by diagrams or by providing suitable experiments. The problems should occur in real or visualized situations; they should be short and involve only easy numbers and easy manipulations. The problems should be couched in words suited to the child's vocabulary and language ability; many children who seem unable to tackle problem questions fail because of inadequate understanding of the phrasing of the question. It is advisable to avoid problems of the inverted type, that is to say, problems in which the information obtained in the answer would, in general, be more easily obtained than the information forming the data of the problem.

INDIVIDUAL, GROUP AND SECTIONAL WORK.

In the systematic study of arithmetic it is essential that one stage be thoroughly mastered before subsequent steps are built upon it. It is also of great importance that the interest and enthusiasm of the more intelligent pupils are maintained consistently, and that unnecessary drill and drudgery are not forced on an intelligent child after an adequate degree of efficiency has been achieved. The teacher of a class of children of varying ability is, therefore, faced with the problem of persisting with a topic until it is generally understood, without killing the interest of the better pupils. This problem must be solved by each teacher for the particular class to be dealt with, but, in most cases, some system of grouping will probably be adopted. The teacher must of necessity study the class and grade the children carefully for arithmetic, bearing in mind that the group in which a child is placed may vary from subject to subject, a child not necessarily being kept rigidly to one group even in arithmetic lessons. Ideally each child should be studied as an individual: with large classes this is not practicable on any large scale, but it is often easier to consider the individual in relation to the group than in relation to the class as a whole. The object of dividing a class into groups is to help the teacher to meet the requirements of each child as efficiently as possible. Frequently a new topic can most conveniently be introduced to the class as a whole and some of the earlier working on the subject be carried out by all the children. A stage will, however, be reached when the class has actually divided itself into groups. Speaking in general terms, the teacher may, for instance, find that one group of more backward children will still be trying to understand the nature of a problem to be solved, while another group has passed this stage and is building up the steps in the solution, and a third group has perceived the whole problem and method of solution and is finally calculating the numerical work.

The organization of a class in groups is simplified by the preparation and selection of carefully graded examples and the skilful use of text-books. With all children the development of a new process should start with very simple examples, the solution of which is obvious, gradually working in easy stages from these beginnings to questions which are themselves interesting and also give a sense of achievement to the child who has solved them. The work must be presented in such a way that the child is confident of his ability to find a solution and also able himself to see that he is making progress in his work. Where a new process can be taught by allowing the children to discover it for themselves by means of carefully graded examples or experiments, the teacher will be able to devote some time to individuals needing special help. In all cases individuals must be considered, the division of the class into groups being a means to that end.

THE PROJECT METHOD.

The necessity of logical development in the teaching of arithmetic and the importance of completely mastering one process of calculation before a new step is attempted, combine to make this subject traditionally one of the most abstract and academic in the curriculum of the Primary School. Although it is true that some children find satisfaction in the correct working of sums that are either purely mechanical or are solutions of quite unreal problems, it is now realized that children in the Primary School reach a better command of the Arithmetic they will need to use in adult life if they are trained in the early years to work out calculations which they themselves require to enable them to make something or to understand a real situation. This is the essence of the project method. Where a class or group of children need to measure, to count or to reckon before they can carry out an interesting scheme such as the construction of a model railway station, the printing and selling of tickets for a school entertainment, or the making of a hutch for a class pet, they will be using their number knowledge as a means to that end. For the majority of children this is the best stimulus and should be used whenever the occasion offers. There will be some interests, such as transport, newspapers, the Post Office, which will be deepened and enriched if quantitative comparisons are made and experiences requiring measuring, counting, weighing and trading are included. These types of project may well be used to give meaning to arithmetic and to give the children practice in calculations through pleasing and natural occupations. At any period in the Primary School the appearance of a strong interest of this kind should be welcomed and fully used. There are, however, two limitations to the amount of arithmetic that can be so taught. The first is the obvious logical difficulty that a numerical process cannot be learnt unless the necessary preliminary processes are known. No attempt should be made to force the children beyond the steps that they can happily and satisfactorily manage at this stage, no matter how excellent an opportunity the project appears to offer. The second is less obvious but of equal importance; if a project is used as the pretext for introducing arithmetic which is not in fact necessary to it, nor is felt by the children to add to its interest, or understanding, then the project fails in its main purpose, namely, to provide a motive for learning or a suitable setting for occupations spontaneously enjoyed by the children.

If projects and interests are allowed to develop at intervals during the Primary School course, such arithmetic as is essential to them will be well learnt, but the committee would urge the importance of keeping a logical scheme intact and of providing regular opportunities of practice in arithmetical skills.

Apparatus and Materials.

In the Nursery School and the Infant School the children will in most cases have been helped in their earlier arithmetical work by the free use of carefully prepared and tested apparatus. For the Junior School there is not available such a profusion of sets of manufactured apparatus suitable for children up to 10+; the teacher is therefore called upon to use his own inventive powers and initiative in order to provide material aids to learning. It is not proposed, here, to do more than mention certain guiding principles and to make one or two suggestions for apparatus which may be adapted by teachers to meet their own requirements.

In previous chapters certain suggestions for apparatus have been made. The apparatus suggested is designed either to clarify or to improve the working of a particular process, or to provide concrete material with which to introduce a new process. Apparatus may also be used to provide material for the solution of concrete problems, and also for drill work for specific weaknesses in backward children. In each case the apparatus is designed and used for a specific purpose, the children are aware of its usefulness for that purpose, and know why they are using it. In the first place it must be stressed that ordinary things met with in the everyday life of the children can provide most efficiently almost all the material necessary. The use of specially prepared apparatus, never seen outside arithmetic lessons, tends to carry even the applications of arithmetic away from the practical into the realms of academic uselessness. The introduction into the classroom of scales and weights, tape measures, boxes, omnibus tickets and fare tables, time tables, Meccano, boxes, nails, local maps and such real things familiar to the children, does much to remove the halo of mystery which still circumscribes some of our work.

These everyday articles are introduced for three distinct reasons. They may help to illustrate, or give practice in, an arithmetical process which a child needs to master, or they may be used to exercise the children in the application of an arithmetical process already known or, thirdly, the arithmetic lessons may show how the articles themselves may be used more efficiently and for a wider variety of purposes. This last use illustrates particularly the practical value of pure arithmetic regarded as a tool, it being presumed that the increased efficiency in the use of the materials follows proficiency in number work.

In addition to the articles mentioned above, but within the same general class, models, such as the clock-face, may often be used with considerable value.

Of the specially designed apparatus which may be used in the teaching of arithmetic we mention first the mathematical instruments such as a ruler, set-square, compass, protractor and so on. Many children in the Junior School will make use of some, at least, of these, but insistence on the accurate use of them for precise geometrical work will, in general, be deferred to the Secondary School. The following examples of special apparatus may serve to give ideas to teachers who have need of special methods for helping children who need more help in certain processes.

(*a*) Cards for the tens-notation (see Chapter III, page 18). The number 426 means four hundreds and two tens and six units, that is the sum of the numbers on these three cards:

$\boxed{4\;0\;0}\!>$ $\boxed{2\;0}\!>$ $\boxed{6}\!>$.

These may be superimposed, giving $\boxed{4\,|\,2\,|\,6}\!>$. Removing $\boxed{2\;0}\!>$ leaves $\boxed{4\;0\;6}\!>$ and so on.

(*b*) "Dominoes" for practice in tables at all stages. For example, the five dominoes $\boxed{2 + 6 : 3 + 7}$, $\boxed{8 - 1 : 3 + 5}$, $\boxed{9 - 4 : 6 - 2}$, $\boxed{5 + 5 : 2 + 3}$, $\boxed{10 - 6 : 5 + 2}$ must be sorted by the children and built up in a row so that adjacent ends tally, thus: $\boxed{8 - 1 : 3 + 5}$ $\boxed{2 + 6 : 3 + 7}$ $\boxed{5 + 5 : 2 + 3}$ $\boxed{9 - 4 : 6 - 2}$ $\boxed{10 - 6 : 5 + 2}$. If these are carefully prepared so that there are no duplicates and the open ends also tally, then the child can find out for himself whether he has made a mistake or not. This method can be used for isolated backward children or for revision with a group in the class. By preparing sets with suitable work on them, the dominoes may be used throughout the Junior School.

(*c*) Strips of card with numbers arranged on them as in the accompanying figure may be used for varying a set of calculations for practice. They may be used for practice with any of the four rules.

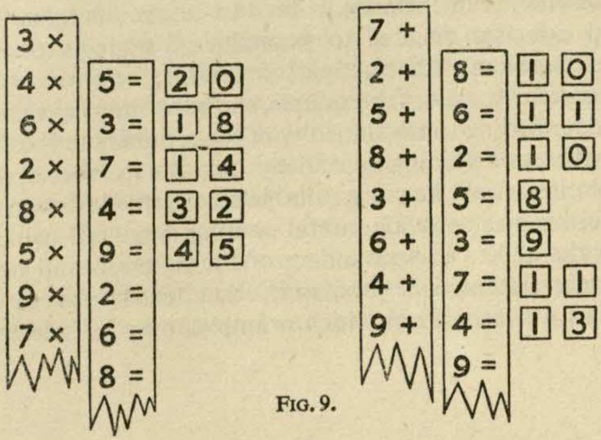

Fig. 9.

(*d*) In work on areas it is suggested that transparent material ruled in squares, to lay on top of a given figure whose area is required, would often give an easier exercise than copying the figure on squared paper.

CLASS NOTE-BOOKS AND RECORDS.

It is strongly urged that each child keeps an individual record book. When a table has been worked out by a child and when any useful fact has been discovered they should be recorded and in subsequent work requiring their use the pupil will refer to his own record book. This must be kept up to date and the teacher must see that all records made therein are correct. There is every reason for allowing a child to refer to correct records rather than insisting upon a memorized result; if memory is false then the mistake is in danger of being perpetuated. A record of the pupils' own calculated results is to be preferred to a printed book of tables because the record book should only be compiled as the individual child understands, masters and finds for himself the results recorded. A child incapable of compiling for himself a table of sevens cannot refer to a printed table with the confidence given by understanding. The record book must, of course, be distinct from the ordinary exercise book.

THE USE OF TEXT-BOOKS.

Since each different class has its own problems and its own special requirements, a text-book written for use in hundreds of different classes needs to be used with discrimination by the class teacher. For instance, some text-books contain carefully graded exercises devoted to examples of a single process or type of problem. The method of grading, however, may not suit a particular class; for example, the book may have examples graded according to the difficulty of the numbers in the calculations whereas a grading dependent upon the number of steps in the solution would be more suitable at a particular stage. Some text-books may provide useful reading material which will enrich the child's understanding of the function and necessity of certain arithmetical processes, other text-books are of use solely as a source from which examples may be selected. As long as it is regarded as a useful servant and not as a tyrannical master, the text-book can be most helpful.

MARKING AND CORRECTIONS.

It is of vital importance in Arithmetic that the work of the class be carefully checked by the teacher. This does not necessarily imply that every calculation made by each child must be personally marked by the teacher at once; in fact in Arithmetic more than in any other subject the child can himself discover whether he is right or wrong in his answer. The importance of checking and going through the work lies in the indisputable fact that working must be accurately performed or it is useless. It is probably safe to say that if a child makes a mistake either in recalling a table or in procedure, then it will take four or five correct workings to overcome the tendency to repeat the error. This was known many years before modern psychologists subjected the formation of habits to scientific study, and led to the traditional system of writing out the corrections of mistakes ten times. Some writing out of corrections is admittedly necessary, but it is frequently better to set another example, requiring a repetition of the working previously wrong. This allows a slightly different approach to the crucial point and the correct working should be performed without difficulty. Should the error be repeated then more drastic steps must be taken to correct it. Apart from badly supervised previous working which has permitted simple errors to become serious habits, the inability of a child to overcome a mistake which he knows he is liable to make must mean either that he is being set work in advance of his mental capabilities or that he has some psychological defect which calls for specialist treatment. A child who has lagged behind the rest of the class must be treated very carefully and not pushed beyond his capabilities. The serious case of a child with definite number disabilities is really outside the scope of this Report but an appendix has been included giving a bibliography which may help the teacher. One of the chief responsibilities of the teacher of arithmetic is the careful scrutiny of the work of any child making frequent mistakes so that the precise nature of the mistake may be discovered and remedial measures carried out.

APPENDIX I

THE BRITISH MEASURES: HISTORICAL NOTES

The traditional British system of measures is derived from the measurement of the human limbs. The story of the development begins in pre-history and is one of evolution rather than of invention.

Different districts in one country and different countries introduced their own separate units of measurement. The British measures have clear connections with the early European measures (*not* the Metric system), with Rome, Greece, Egypt and the East. The variety of measures in these different places at different times is a variety of name; but the origins seem to have been the same. They are human measurements and this points to the idea that the early English measurements were adopted as natural aids and were found afterwards to be similar to those in older civilizations. The final selection of the standard measures is the result of centuries of controversy. The history of their gradual development in England may be traced in the Laws of England from Anglo-Saxon times. It is full of intricate detail and there is not space here to do more than outline the history of a few of the measures. It is interesting to note European and Greek-Asiatic influences and the changes of language in which the laws that dealt with these units were written.

When the Anglo-Saxons arrived in England they did not impose measures, they adopted what they found, no doubt because they were the natural measures. In England from 560 to 1488 the legal language varied from Anglo-Saxon through Latin, Norman-French, and French to English. After 1488 the laws were written in English. As the language became stabilized so the measures became stabilized.

The basis of the measurements we use to-day is still the human length and the measures of England are the result of an ordered democracy with its foundations in agriculture and commerce rather than a mathematical dictatorship imposed by legislators.

The metric system is enforced by law in France, but the market-man sells his goods by the pound (*livre*) when the gendarme's back is turned. Germany's recent attempt to divide the right angle into 100 degrees failed because the degree which is 1/360 and not 1/400 of four right angles is a unit of measurement with a natural origin; and sailors all over the world use the nautical mile, a world measurement connected with the 360 degrees division, which dates back to the times of the ancient Chaldean astronomers.

The end of this appendix is concerned with a summary of historical data. This is of importance to the teacher as a background. The historical information may be introduced into lessons when suitable. Some of it is beyond the understanding of Junior School children and should be introduced later; but the natural measures and their standardization form a subject that can be developed in practical lessons.

Examples.

(1) Several children and the teacher could measure the length of

- (*a*) A wall as so many arm-stretches, i.e. the distance between finger-tip and finger-tip of outstretched arms.
- (*b*) A table in hand spans.
- (*c*) A floor plank in foot lengths.

The results in each case vary. The children should be told that these are the measures that were used long ago, and the difficulties which naturally arose in buying and selling goods could be discussed.

(2) The difference between the uses of capacity and weight could also be discussed. Long ago goods were sold to a larger extent than they are to-day by capacity. But the sale of many goods by capacity led to disagreement and often to cheating. Think of using the quart measure for the sale of biscuits! Some goods are sold in either way nowadays, gooseberries for

instance; but the capacity measures are now generally used for liquids and a few dry goods only because we now have accurate weighing machines and it is usually fairer to measure by weight.

The comprehension of these measures as practical aids should command more attention than the working of reduction sums.

Unfortunately the ramifications of present-day life do not provide such a wide field of personal use as did the simpler lives of earlier people; a simple account of the historical background will fill this gap. The very fact that they are based on measurements connected with the human limbs means that children will comprehend the use of them by learning to use their own limbs for measuring. This provides a field of physical activity which children in a Primary School need. A more extensive study of the history would interest some children in a Secondary School, especially in a rural area.

The following short summary may prove useful to teachers.

LENGTH.

The Ancient Measures.

The fathom.—The height of a man or the distance between finger-tip and finger-tip of outstretched arms (about 72 inches).

The cubit.—The length from elbow to finger-tip (about 18 inches).

The span.—The length from thumb-tip to little-finger tip of outstretched hand (nearly a half-cubit).

These are some of the rough-and-ready human measures.

The mile.—The world measurement.

At least sixty centuries ago the Chaldean astronomers divided the circumference of the earth and of circles into 360 degrees each of 60 parts. The length of arc of a sixtieth part of a mean degree is the meridian or nautical mile.

The Greek geometers divided this meridian mile into fathoms and then feet.

The old English Measures.

There are many overlappings of names. For instance, *Yard* and *Furlong* sometimes meant length and sometimes area.

Yard—(O.E. gerde or yerde. A. S. gyrd or geard. Fr. verge.) This seems to have been used in two ways, as a length varying from a wand (about the present-day yard) to a measure of 24 ft., and secondly, as a measure of land surface, which again varied in size, cf. vine *yard*, farm *yard*.

Its use to-day as a long measure still preserves its connection with the human height. 1 yd. = ½ fathom = half of a man's height.

Foot.—This name is no doubt from its approximation to the length of a man's foot.

Inch.—The combined length of three barleycorns.

The Roman foot measure was approximately the same length as the English foot of to-day. It was divided into 12 uncia. Uncia means a thumbnail breadth or a twelfth part (cf. ounce in weight).

Rod.—This varied from about 16 ft. to 24 ft. This was evidently the length of a *rod* or *pole* in England. In France it was a "perche" (cf. Eng. perch). Spears in France and lances in Scotland were as long as six yards and may have been used as a standard length. It may have been the length of an ox goad, but these are believed to have been about three yards long. (Cf. rood in *land measure.*)

Furlong.—Meaning "furrow long," a measure for ploughed land. (See *land measure.*)

Mile.—The old English mile was 5,000 feet. (The Roman mile was also 5,000 ft.)

Meridian Mile.—The nautical mile for all sailors. This is the Egypto-Greek mile of 6,080 ft. and dates back to the Chaldeans.

Chain.—Nearly three centuries ago Edward Gunter introduced the chain measure. This is a surveyor's chain of 100 links. The equation of 10 chains to 1 furlong is the only decimal item in the table.

Land Measure.

The Domesday Book set forth a valuation of land for purposes of taxation, and there is not the smallest indication that there was any intention to interfere with the existing measures. In early times the peasant could estimate the area of land at sight, just as a farmer does to-day. His units were naturally based on the furrow length and the rod. When ploughing heavy fallows three or even more yoke of oxen were used, and a furrow or furlong of one rod breadth would be done in one "shot." The oxen would then pause for breath and the ploughman would measure with his goad the next rod breadth. The oxen would turn for the next furrow. One of these ploughed furrows gives the rood land measure, and four of them give the acre. Ten of these oblong acre strips gives an area of one furlong square.

This square furlong is probably our original unit of land measure. It was called the ACREME.

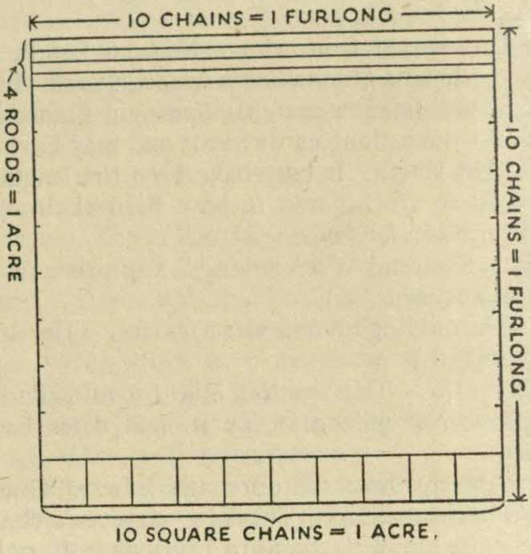

FIG. 10.

The square furlong or *acreme* or 100 *square chains* or 40 *roods.*

The later introduction of the chain measure by Gunter decimalized the land measures without disturbing them.

The other units in the area table have obviously arisen as squares of the length measure.

WEIGHT.

Commerce, after the most primitive stage of bartering goods by counting them, necessarily led to two other forms of measure —to WEIGHT and CAPACITY. Measuring by the capacity of a hand-full or of a customary basket or pot, was superseded by weighing their contents as soon as a weighing machine had been devised. The Auncell was a primitive weighing machine used in the market-place. It consisted of a bar balanced on a fulcrum. Goods were hung on one end and weights were adjusted on the scale marked on the other end. A stilyard or steelyard is of the same type. This type of machine was inaccurate and encouraged cheating. The goldsmiths used a balance with two scale pans. The name *avoirdupois* refers to the use of this type of balance for weighing, i.e. a weight lying in a scale pan to balance goods in the other scale pan. Statute recognition of avoirdupois dates from the time of Elizabeth, but mention is made of it before this. It was sometimes spelt "haberdepase."

THE UNITS OF WEIGHT are based on the ancient TALENT.

A talent (= 1 cubic foot) was used as a unit of weight and of capacity. In Egypt the talent was the weight of a royal cubic foot of water. It was divided into 125 librae of 12 unciae each.

The uncia was thus 1/12 of the Roman pound (libra) and 1/12 of the Roman foot. (See "inch" in *length measures.*)

An ounce is still 1/12 of a Troy pound. (See later under coinage.)

Abbreviations.

Lb. for pound is obviously a shortened form of libra. (Latin pound.) Oz. for ounce may be o with a sign of abbreviation, later mistaken for z. There was an old Spanish coin called an "onze."

The story of the avoirdupois table of units seems obscure, but there is good reason to believe that the Roman standard passed to Britain and the libra (pound) was raised to 16 ounces. This seems to have been an English standard of weight before as after the Norman Conquest.

The Early Plantagenet Weights.

A stone of 16 pounds.
A wey of 16 stones.
A hundredweight of 100 pounds.
A ton of 20 hundredweights.

To a large extent it seems to have been a system based on sixteen, just as capacity (see later) was based on eight. The stone of 16 pounds has survived perhaps in a half form in the butcher's stone and the farmer's score each of 8 pounds. During the period from Edward I to Edward III, the weight table seems to have been unnecessarily revised. The hundredweight (centena) was raised to 108 pounds and then to 112 pounds. The relation of the hundredweight to 8 stones of 14 pounds may have been arrived at through continual halving.

$\frac{1}{2}$ quarter = 1 stone = 14 pounds.

The extra 12 pounds over the original hundredweight may have been legalized in order to prevent the giving of underweight.

Tonnage of Ships.

The tonnage of ships is a measure of capacity not of weight. 10 quarters of wheat = 80 bushels = 5,000 pounds weight. This bulk of wheat measures 100 cubic feet. This 100 cubic feet is the cargo ton, and is the unit adopted in all maritime countries. A ship of 2,000 tons register is of a capacity 200,000 cubic feet below decks.

CAPACITY.

As we have said the talent—a vessel of one cubic foot—afforded the standard for units in capacity as in weight.

The Roman talent—approximately 1,000 Roman ounces = $62\frac{1}{2}$ lb. (16 ounces to the pound.)

There is reason to believe that this cubic foot was our original wine unit—the wine bushel. The wine gallon, originally $\frac{1}{8}$ wine bushel, was gradually raised until under Henry VII it contained about 8 pounds weight.

As the capacity measures were thus related to weight, difficulties arose owing to the different specific gravities of commodities (cf. wine and wheat). The bushel of wine would be smaller than the bushel of wheat. The variety and difficulties led eventually to standardization and in 1884 the tables were unified. A single gallon is now equivalent to 10 lb. water at an ordinary temperature. The corn gallon gives a weight of 64 pounds to the bushel—so that 1 pound wheat corresponds to about a pint.

The capacity measures still preserve the 8 relationship.
The weight measures still preserve the 16 and 8 relationship.
The gill still varies (2 or 4 to the pint) with locality.

TIME.

In calendar and clock times we preserve a connection with the most ancient astrologers. The primitive conceptions of time were a day (between sunrise and sunset) and the lunar month taken as 30 days. Thus the twelve lunar months gave a calendar of 360 days.

The ecliptic is the sun's apparent path in the heavens. The ancients divided the circle of the ecliptic into 360 degrees of arc, each division to correspond to a day. The degree was subdivided into 60 minutes (minute divisions) and the minute divided into 60 seconds (secondary divisions). They used this as a method of division of other circles. But the sun takes 365 days, 5 hrs., 48 mins., 46 secs. to complete this apparent motion. We have preserved the 360 division of circles as it is a more convenient number than 365.

The calendar is adjusted by distributing the 5 days among 5 of the months, and by adding four times the extra hours, minutes and seconds in one day to the February of every fourth year (leap year).

But the number of days in each month is inherited with small adjustments from the calendars of Greek-Asiatic peoples.

The Roman year began in March.

The English year began on Lady Day, 25th March, until 1751. The original names for the months were:

Martius	from Mars.
Aprilis	from Aphrodite.
Maius	from dii maiores, the elder gods.

Junius	from dii juniores, the younger gods.
Quintilis	the fifth month, afterwards Julius.
Sextilis	the sixth month, afterwards Augustus.
Septembris	the seventh month.
Octobris	the eighth month.
Novembris	the ninth month.
Decembris	the tenth month.
Januarius	from Janus or Dianus (the sun god).
Februarius	from februum, the expiation month.

The week with seven days is also of astrological origin—each day ruled in the first hour by one of the seven "planets" (pre-Copernican system), Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon. Some of our names are taken from the names of the Graeco-Roman gods and some from those of the Norse mythology.

Saturn	dies Saturni	Saturday
Sun	,, Solis	Sunday
Moon	,, Lunae	Monday
Mars (Tiw)	,, Martis	Tuesday
Mercury (Woden)	,, Mercurii	Wednesday
Jupiter (Thor)	,, Jovis	Thursday
Venus (Frigu)	,, Veneris	Friday

With the invention of clocks and the progress in astronomical instruments, came the introduction of the mean solar day.

This is measured from noon, which is the time at which the sun crosses the local meridian.

As the solar day varies, clocks are regulated to the average (mean time) of all the solar days in the year.

TROY WEIGHT AND COINAGE.

> "That the measure of a Bushel contain 8 gallons of wheat, and that every Gallon contain 8 lb. of wheat of Troy weight, and every Pound contain 12 ounces of Troy weight, and every Ounce contain 20 sterlings and every Sterling be of the weight of 32 Corns of wheat that grew in the midst of the ear of wheat according to the old law of the land." (Statute of Henry VII, 1496.)

This is an interesting record of the use of Capacity and Troy weight together.

The Troy pound is the standard of weight for precious metals. It was used alongside the Avoirdupois weight in medieval times. Troy weight had a longer and more stable history because it was the money standard weight. The name is derived from a foreign medieval coin, the marc of Troyes, but the connection is only one of name.

Like avoirdupois the weight was eventually standardized but the standardized table was, with small adjustments, the same as it had been for centuries.

The Parliamentary Committee of 1758 established a convenient connection between the two so that

1 Troy pound = 5,760 grains. 1 Avoirdupois pound = 7,000 grains.

(24 grains weight = weight of 32 grains of corn.)

TROY WEIGHT.

1 pound = 12 ounces
1 ounce = 20 pennyweights
1 pennyweight = 24 grains

THE ENGLISH COINAGE.

Peel quoted the following evidence of an accountant in the House of Commons in 1819:

"He was required to define what he meant by the pound. His answer was, 'I find it difficult to explain it, but every gentleman in England knows it.' The Committee repeated the question, and the accountant answered, 'It is something that has existed without variation in this country for eight hundred years—three hundred years before the introduction of gold.'"

The history of the English pound begins with the history of the English penny. From Anglo-Saxon times to the eighteenth century the pound was based on a Silver standard. In the eighteenth century the Gold standard established itself because of the imports of gold in that century. During the centuries of Silver standard one pound (Troy) of silver was minted into 240 silver pennies. These coins were struck from the eighth to the nineteenth century. They diminished in fineness from 24 grains [*circa* 991] to 7·27 grains in 1816.

It is possible that King Alfred used the present-day Troy pound as a Silver standard. William the Conqueror established the Tower Pound, which was a mint pound 6¼ per cent lighter than the Troy pound. This was to enable the Tower mint to coin 240 pennies from less than the Troy pound of silver brought to them by merchants. The merchants received their 240 silver pennies and the surplus silver was kept by the mint partly to pay "seignorage" to the King.

The coins issued were usually from an alloy of 9·25 per cent silver.

Base coinage was frequently circulated but the Norman and English merchants jealously maintained "The Ancient Right Standard of England" and it did not finally disappear until 1920. If we neglect one or two attempts to issue smaller coins (groats) and one attempt to issue gold, the silver penny was the only coin for at least 500 years.

The pound had sub-divisions for accounting purposes only.

The name "scilling," meaning "a piece cut off," was given by the Saxons to pieces of broken silver which they threw into the scale to make up loss of weight when payment was made by weight. Later the value was regularized to mean a definite number of pennies, at first four or five and later twelve. William the Conqueror preferred the shilling of 12 pence.

A short time after the Conquest, the Roman Monetary System favoured by the ruling Norman classes became the coinage of England.

ROMAN MONEY TABLE.

1 *l*ibra (pound) of 20 *s*olidi
1 *s*olidus of 12 *d*enarii
hence £ s. d.

It is interesting to note that the stroke /, as used in 1/3, is called a solidus.

Until the fourteenth century denier (denarius) was used officially to mean pennyweight as well as coin by tale. When silver was weighed in mass it was reckoned in pounds, shillings, and pence.

Early in the twelfth century the penny was called a "sterling" and "sterling silver" ultimately became the silver of commerce in a great part of the world. For small transactions the people cut the penny into halves and quarters and even smaller fragments. Hence the names farthing (fourth) and halfpenny. These shillings, halfpennies and farthings did not exist as coins.

There is not space here to deal with the numerous obsolete coins. Gold pennies and other gold coins were issued intermittently from the thirteenth century. Silver farthings and halfpennies were issued in the fourteenth century. It was a general practice to clip pieces off coins and these light coins were for centuries in general circulation. Two events in the seventeenth century contributed to the reform of the light coinage. In 1663 the French experimenter Pierre Blondeau was engaged to set up his machines in the Mint and issue coins with a milled or ridged edge. This circumvented the coin clippers. Secondly, the mints had been making exorbitant charges for recoining worn coins and bullion; in 1666 an Act provided that any person who brought bullion to the mint should have it assayed, melted, recoined, and for every pound weight of standard metal should receive a pound weight of coins without charge and for baser or finer metal in proportion. About this time an issue of farthings and halfpennies containing nearly their face value of copper was made from the mint. These copper coins were later often made of cheaper alloys.

THE PRESENT-DAY COINS.

The Soveraigne or Double Ryal was a gold coin first issued by Henry VII.

The silver shilling (or Testoon) was first issued in 1504, the threepenny piece, sixpence, half-crown and crown in 1552, the florin in 1849. The last silver pennies were issued about 1816.

The Coinage Act of 1870 standardized the gold coins (sovereign and half-sovereign), the silver coins and the bronze coins, penny, halfpenny and farthing, and stated the types of alloy to be used.

The next important event in the history of coinage was the Great War of 1914. During the week before 6th August, 1914, there was a general fear that the banks would refuse to pay out gold. This had set some people hoarding the metal and there was a genuine fear that this might spread. The Currency and Treasury Bank Notes Act was passed in a single day on 6th August, 1914. This empowered the Treasury to issue notes for £1 and 10s. to be current in the United Kingdom as fully as sovereigns and half-sovereigns. The financial crisis that followed this led in 1920 to the final collapse of the "ancient right standard" silver of England which had been in use with one interruption since the Conquest. "Silver" coins were issued of fineness 500 parts in 1,000 instead of 925. The metal in them was now worth much less than their face value; they had become simply tokens of a certain fraction of £1. In 1947 cupro-nickel coins were issued and the silver coins gradually withdrawn.

Here we must leave the English penny and pound. The penny, six of which at the Conquest would have bought a bushel of wheat, will now only buy a very small loaf. The pound, which Domesday Book recorded as the year's rental of a hundred-acre holding, will now scarcely pay a week's rental of two unfurnished rooms on the outskirts of London.

APPENDIX II

THE SELECTION EXAMINATION AT 11

In drawing up this memorandum on Primary School arithmetic, it was felt that the examination at 11+ could not be ignored. With a view to helping them in their analysis of the Primary School course, the Committee studied some two hundred different examination papers set during 1937 from areas throughout England. From a study of these papers it was found that such a wide variety of requirements existed in the different areas that it is difficult to generalize about the examinations as a whole, and it is realized that many of the points mentioned below have little value to individual teachers, but they are included here because it is apparent that much serious attention could be paid to the problem of the Selection Examination. One of the first points to be raised is the purpose of the examination. This aspect may be stated briefly under two heads:

(1) Where the examination is set by the Secondary School the syllabus for the examination largely governs the course in the 10 to 11 year in the Junior Schools submitting candidates, and in some cases it seems probable that the examination is designed to ensure that the children have been given what is considered, by the Secondary School, to be an adequate groundwork in certain specific parts of the work. In other cases there is no evidence in the papers that any influence on the work of the Junior School is intended.

(2) The examinations in all cases seek to establish an order of merit amongst the candidates. This may be regarded as the primary reason for holding the examination. The ability looked for, however, varies so widely that it is interesting to note one or two points. In about a third of the papers the arithmetic was tested almost entirely by setting problem-type questions. The computation involved in many cases was quite

subsidiary to the ability to unravel the problem. In roughly another third of the papers all questions were straightforward number work in which speed at mechanical calculation seemed the governing factor. The remaining group either set two papers, one of each type, or gave mixed papers. It is interesting to note that such a wide difference of opinion exists as to the relative merits of problems and direct calculations as a means of assessing ability. One feels that this question might be studied carefully because upon the attitude of the examiners to the best method of testing depends very largely the emphasis given in arithmetic lessons during the very important years preceding the examination.

In connection with the dependence of the Junior School syllabus on the requirements of the Selection examination certain specific points may be raised. Outstanding among these is the attitude towards decimal fractions. Of all the questions analysed only 3 per cent involved any knowledge of decimals. The few questions included varied widely in the range of knowledge required. From a simple addition of three numbers such as 31·24, etc., the questions varied in difficulty to a question such as dividing a number of inches by 39·37 and expressing the answer in metres *correct* to two decimal places. Only one or two isolated questions required any knowledge of the metric system.

Several papers required the children to find the areas of triangles and one paper included a question on the area of a trapezium (printed on the examination paper). This obviously imposes some formalized mensuration on the children of certain neighbourhoods which is not required from other children.

The Committee was interested to note that in one area from which papers were obtained, the head teachers of Junior Schools sending on children to a single Secondary School have drawn up an agreed syllabus which ensured that all children in the locality had been taught to perform certain basic operations in the same manner. For example, the same method for subtraction and the same manner of setting out division sums are used throughout the area. Whilst the Committee does not wish all teachers to be required to teach everything in a standardized way it is appreciated that in an area which feeds a single Secondary School it must be very useful to agree to some standardization within the area.

With regard to the papers themselves, as distinct from their immediate repercussions on Primary School work, several points may be mentioned. The subjects introduced into the questions seem on the whole far more closely related to the interests of the examiners than of the children. Almost exactly one-third of the questions had money as their principal topic. Bearing in mind that children up to eleven years have little opportunity of spending very much money and that none of them is likely ever to have handled many pounds, this proportion seems rather high, especially when compared with a total of 8 per cent in which units of length figured. Ten per cent involved weights, 2 per cent time, 3 per cent speeds and 3 per cent capacity. Of the 12 per cent which were related to "shopping" some were very unreal. For example, "I pay 8s. 6d. more for 14 yards of material than for 8 yards, what is the cost per yard?" Several sums of this type were included, all representing very unreal problems.

The examination of the papers revealed wide variations in two other important matters: the complexity and heaviness of computations that can be expected from children of 10–11, and the number of consecutive logical steps that they can take to solve a single problem. Both of these raise questions about the mental development of children that still need further investigation, but it seems clear that in some districts demands are made upon children that could wisely be postponed. Another point noted in some questions was that they involved the use of many different units; one sum involved square inches, square feet, square yards, acres and miles.

On the other hand many of the papers give a useful lead to teachers in the selection of real-life situations to which arithmetical knowledge, at this age, may be applied. One interesting question, for example, quoted "Postal Information" from an official source, gave a list of the weights and addresses of certain letters, two being "via Air Mail," and asked for a list of the stamps required. From amongst the many questions we were able to see, a number were really very interesting and helpful.

It is very difficult to state any precise conclusions with regard to the examinations as at present set. They influence the work of Primary Schools in different districts in widely different ways. Individual papers often give a very one-sided test. Some papers invite "cramming," whilst others seem designed to counter it.

It is this variety in standard and form of paper which prevents this examination from exerting the stabilizing and wholesome influence on current arithmetic schemes that well-devised papers could achieve. Teachers are not confident that work on the lines and within the limits they believe to be sound, will prove to be sufficient to give the children a fair chance in this important selective test. Though complete uniformity is undesirable, a general acceptance of a scheme based on children's known abilities and interests seems long overdue.

One outstanding fact, evident to all experienced examiners, is that it is useless to set an examination paper which will not give a good spread-over of marks, and where the intelligent candidates will not, on the whole, show up well. As long as some Junior Schools invalidate the examination papers by coaching too intensively for them, some examiners may continue to set a wrong type of paper. A frequent experience in a Grammar School is that primary scholars are too advanced in the technique of getting certain types of sum correct, yet have little intelligent understanding of fundamental arithmetical ideas.

It is important to observe here that if Primary School teachers would have the courage and interest to reform their syllabuses they might, in time, dictate a better policy for some examining bodies, who would then set an examination which would select the intelligent children and provide the Secondary Schools with children suited to their courses. Most Secondary Schools would welcome a change to a policy of understanding fundamentals rather than acquiring skilled technique in certain operations, some of which are too advanced for children of that age.

The examination is often mistrusted by both Primary and Secondary Schools. Most Primary School Heads, because of their personal interest in the candidates' chances of a Grammar School education, would not adopt the drastic policy of disregarding the examination, however strong their opinions may be. For an examination at this age to be satisfactory, it is essential that teachers and examiners be in sympathy with each other's problems and ideals and a closer co-operation between them would undoubtedly be profitable both to the schools and to the children.

APPENDIX III

BIBLIOGRAPHY

The following list of books was compiled to be a guide to teachers interested in the results of recent investigations and in the diagnosis and cure of specific arithmetical weaknesses. A few books have been added for the usefulness of their suggestions for practical work and practice games. Brief reference is also made to books on weights and measures.

(*a*) THE RESULTS OF RECENT INQUIRIES

Studies in Arithmetic: The Scottish Council for Research in Education (University of London Press).

Research in Arithmetic: G. T. Buswell and Others; Twenty-ninth Yearbook of the National Society for the Study of Education (University of Chicago).

Adjusting the School to the Child: C. Washburne (World Book Co.). Chapters II and III.

(*b*) DIAGNOSIS AND REMEDIAL TEACHING

Diagnosis of Individual Difficulties in Arithmetic: F. J. Schonell (Oliver and Boyd).

Schonell Diagnostic Arithmetic Tests: F. J. Schonell (Oliver and Boyd).

Diagnostic and Remedial Teaching in Arithmetic: L. J. Brueckner (J. C. Winton Co., Chicago).

Corrective Arithmetic: W. J. Osburn (Houghton Mifflin Co., Boston).

Diagnostic Studies in Arithmetic: Buswell and John (University of Iowa).

(c) SUGGESTIONS FOR PRACTICAL WORK AND PRACTICE GAMES

Beacon Arithmetic Teachers' Manual: C. M. Fleming (Ginn and Co.).

Arithmetic in Action: Brideoake and Groves (University of London Press).

Number in the Nursery and Infant School: E. E. Kenwrick (Kegan Paul).

The Teaching of Arithmetic: A. Monteith (Harrap).

Arithmetic is Easy: M. M. Rogers (Evans).

(d) HISTORY OF WEIGHTS AND MEASURES

Men and Measures: E. Nicholson (Smith and Elder).

The Pound Sterling: Feveayear (Oxford).

British Weights and Measures: C. M. Watson (Murray).